Very young children's understanding and use of numbers and number symbols

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Dedication

I dedicate this work to Rosstam, Sabba and Ella and to the numerous children I have met across the years who have struggled or succeeded with mathematics and inspired me to commence and to complete this thesis.

Abstract

Children grow up surrounded by numerals reflecting various uses of number. In their primary school years they are expected to grasp arithmetical symbols and use measuring devices. While much research on number development has examined children's understanding of numerical concepts and principles, little has investigated their understanding of these symbols.

This thesis examines studies of understanding and use of number symbols in a range of contexts and for a variety of purposes. It reports several studies on the use of numerals by children aged between 3 and 5 years in Nursery settings in England, Japan and Sweden and their understanding of the meanings of these symbols.

167 children were observed and interviewed individually in the course of participating in a range of practical activities; the activities were designed for the study and considered to be appropriate and interesting for young children.

The results are discussed in terms of how they complement existing theories of number development and their relevance to early years mathematics education.

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Table of Contents

List of Tables

Chapter 4

Chapter 5

Chapter 1

The development of young children's understanding of number

Introduction

During the last half of the twentieth century one of the major theoretical approaches to examining young children's understanding of number has, according to Nunes and Bryant (1997), emanated from a belief in the close relationship between universal intellectual development and children's mathematical achievement. Within this field much research, particularly that of Piaget, has focused on children's grasp of logic and their understanding of quantitative invariants such as counting and conservation.

More recently, research has broadened to consider children's problem-solving strategies and errors in mathematics. It also examines the relationship between children's achievements and difficulties and the support available within the culture, particularly in the home and school (Nunes & Bryant, 1997). Nevertheless, the majority of such research focuses on the knowledge, concepts and principles that underpin children's use of number. Much of the work on children's difficulties and errors reports on the problems that children experience when they encounter written numbers and symbols, yet on the whole this aspect of mathematical development has attracted relatively little attention from researchers. This is the area which will be investigated in this thesis.

It was decided to explore the strategies and difficulties that children experience in early number for the following reasons: a) this area is central to the primary curriculum; b) children often experience distress on encountering difficulties and failure in this area; c) there appears to be relatively little published research on younger children's understanding of written (symbolic) aspects of early number and problem-solving, or on the relationship between their conceptual understanding and understanding of symbols.

The thesis focuses on three different domains of number development:

- how very young children make use of cardinal number concepts and symbols to solve problems
- the meanings that young children hold for printed numerals in their environment that convey cardinal, ordinal and nominal information
- the extent to which young children are able to use number and written numerals to represent broader aspects of number, such as age, the time, the date etc.

Linking these different studies is the aim of examining the relationship between the sense that children make of a range of number meanings, their ability to understand their use for a range of purposes in the environment in everyday life, and their ability to make use of oral and written number that is relevant and appropriate to the situation. Finally, the study aims to examine the extent and way in which all these aspects of mathematical understanding may be interrelated in order to address questions related to the means by which number sense is developed in very young children.

The first two chapters examine different aspects of cardinal number and include a review of the relevant literature. Chapter 3 contains descriptions of the empirical tests, and chapters 4, 5 and 6 present their results. In Chapter 7 the results are discussed in terms of how the findings might extend or complement existing theory on children's mathematical development. Chapter 8 reflects upon the design and administration of the studies and Chapter 9 considers recommendations for further research within psychology and the implications of the findings for early years education.

1.1 Piaget's work on cognitive development

Piaget's contribution has been seminal for research into the nature of children's understanding of mathematics and been a common point of reference in subsequent research and critical debate in this field. His work has also been a major influence on Primary education. For these reasons a substantial section is allocated to details of his tests and theory.

Piaget's general theory of cognitive development sets out progressive stages in the nature of children's thinking in which he distinguishes between three kinds of knowledge according to their source and structure: physical, logico-mathematical and social. Physical knowledge is seen as knowledge about objects which can be observed in reality, e.g. colour. Logico-mathematical knowledge is based on abstract, rather than concrete or visible, relationships. Social knowledge is seen as the conventions worked out by people, that is as transmitted from one person or one generation to another. (However, even the identification of physical aspects, eg. colour, and the social conventions of what constitutes a chair or a stool, also rely on categorisation - which is itself a form of logico-mathematical knowledge.)

This chapter will examine in particular Piaget's theory of logico-mathematical development.

1.1.1 The Logic of Number

Piaget's early work focused on young children's ability to grasp the particular principles of pure logic or 'logico-mathematical understanding' as he considered such understanding as underlying and fundamental to a secure grasp of mathematics. Piaget's studies of young children's logico-mathematical development focused on their understanding of the conservation of number, class and order.

Piaget used his test results to illustrate his views on the nature and limitations of children's understanding in the early years of education. He suggested that logico-mathematical development takes several years to attain, a view which has since had an important influence on teachers' expectations of children in and beyond the nursery and infant school.

1.1.1.1 Cardinal number: One-to-One Correspondence and Conservation of Number

Piaget's belief that pre-school children's logico-mathematical understanding is limited led him to view counting at this age as 'just verbalisation' (Piaget 1953), a rote procedure not connected to an understanding of quantity (Dowker & Cowan, 1998). Piaget did not view learning to count as constituting an understanding of number for two reasons: a) learning to count is not sufficient to bring about changes in children's understanding of number e.g. conservation; b) children may develop an understanding of number independently of learning to count Piaget (1952).

One-to-one correspondence

The number conservation task is one of several tasks designed to assess young children's understanding of the principle of one-to-one correspondence (Inhelder & Piaget 1963 cited in Kamii 1994). This involved two tall identical glasses placed before the child and the researcher, and 30 to 50 small beads. The adult asks the child to drop one bead into the adult's glass while dropping one into the child's glass and to continue with this process. Once the child has put around 5 beads into each glass the adult stops the child, asks her to watch, drops one (additional) bead into only the adult's glass. The adult then asks the child to continue with the activity. When a further 5 beads have been placed into both glasses the activity is then stopped - at which point the adult still has one more bead than the child. The child is then asked: "Do you and I have the same number, or do you have more, or do I have more?"

When asked to recall the task most 4 year-olds correctly recall all the observable or empirical facts, but when asked to decide whether or not one glass has more beads most 4 year-olds reply that the two glasses have the same amount "because I can see that we both have the same." A small number claim they have one more than the adult, but can not justify this, basing their judgement on the empirical appearance of the two quantities. While most of the 5 and 6 year-olds are able to logically deduce that the experimenter has one more bead, when they are asked to explain the reason they also cite the appearance of the glasses.

When these are children asked: "If we drop beads all day in the same way, do you think you and I will have the same number at the end, or will you have more, or will I have more?" children aged 5 or 6 differ from 4 year-olds. Some children are reported as making empirical statements such as: "I don't know because we haven't done it yet", demonstrating an accurate recall of the words spoken by the adult - and a literal interpretation of the question.

Conservation of number

Piaget closely related changes in children's responses to number conservation tasks to corresponding changes in their understanding of one-to-one correspondence. He devised further oral and practical experiments which were designed to explore young children's ability to understand 'invariance' or the conservation of number, as he considered this to be a concept fundamental to children's mathematical understanding. Subjects were boys and girls aged between 3 years 6 months and 6 years 7 months. The conservation tests involved inducing or 'provoking' one-to-one correspondence between two sets of objects as a means of observing the extent to which the children make use of correspondence in a spontaneous way.

In tests of provoked correspondence four situations were used. In the first test children are shown six small bottles placed alongside a set of small glasses and the children asked about what is needed in order to have a drink. They are then asked to take just enough glasses, the same number as there are bottles, with one glass for each bottle. Once each glass is placed in front of a bottle, those children who have selected too few or too many bottles are asked: "do you think they're the same?" They are then given further help by being prompted to empty each bottle into a corresponding glass. Once the correspondence is established the six glasses are placed together and the children asked again: "are there as many glasses as bottles?" Where the answer is no the child is asked: "where are there more?'" and "why are there more there?'" The glasses are then re-arranged in a row, the bottles placed together and the questions repeated each time.

Two further tests were given using vases and flowers, egg-cups and eggs. (Piaget 1952) The third test is similar but the child is asked to exchange pennies for a collection of objects. The fourth test is different only in that the children are presented with counters arranged in various shapes and then asked to take out of the box the same number of counters to match the original pattern. If the child correctly matches the shape of this figure, one shape is then changed and the child asked whether there are still the same number of counters. Mistakes are made only by the youngest children i.e. up to 5 years of age.

As shown in Piaget's experiments, the process of assigning a number to an object belonging to a sequence is the 'ordinal' aspect of number, and the step of knowing that the final number can be used to represent the numerosity of the set as the 'cardinal' aspect. This is the link between the ordinal and cardinal aspects of number.

1.1.1.2 Class inclusion

In contrast to simply placing or reciting numbers in the correct order, the concept of cardinal number rests upon the hierarchical relationship of 'class inclusion'. This requires the child to mentally take account of and include all the objects that precede it - for example the eight items when she states that the quantity is the (cardinal) number of "eight". In order to test this Inhelder & Piaget (1967) conducted tests which involved giving children six plastic tulips and two plastic roses of the same size. They are then asked: 'are there more tulips or more flowers?' Most four-year-olds are found to answer: 'more tulips'. When asked 'more tulips than what?' some answer '... than roses'.

For Piaget this indicated that these young children are not able to think about the two sub-sets and the whole of the collection simultaneously; they can only think about the two parts at any one time, but cannot compare it with the whole set until around the age of 7 or 8.

These results also led Piaget to conclude (Greco, Grize, Papert and Piaget, 1960) a key point in his theory, i.e. that language is not the root of logico-mathematical knowledge (in Kamii 1994), based on his view that it is innate structures that result in the logical organisation of children's thinking.

1.1.1.3 Ordinal number: seriation

The tasks involving bottles and glasses that Piaget designed to test for conservation also require children to take into account the ordinal as well as the cardinal aspect of number. When children proceed to arrange both sets of objects side by side so as to match them against each other, they are simultaneously forced to pay attention to order. However, Piaget also went on to design several further tests which would specifically measure young children's understanding of ordinal number.

The seriation experiments presented the child with a collection of between 10 and 19 objects of varying length and asked to re-arrange these according to the ordinal aspect of either ascending or ascending length. i.e in a staircase effect. Having done so, the child was asked questions such as 'how many steps (i.e rods) has this doll still to climb?' and 'how many steps has she climbed already?'

The results show that children do not attain Piaget's Stage III in the seriation test until between one and two years after reaching the corresponding stage in the conservation test, which would suggest that this is due to the demands of combining the cardinal with the ordinal aspect of number. In summary, by the age of 6:6 and 7:7 years, young children move on from reasoning which is dependant on the order in which a group of objects is displayed, to begin to reason more logically and flexibly. Piaget noted that they make use of counting on and counting back, strategies which relate to number itself (i.e in the abstract) and no longer relate solely to concrete objects.

1.1.1.4 Piaget's stages of development

The data from the above experiments were used by Piaget to identify three stages in the understanding and performance of young children:

Stage One: Global comparison without one-to-one correspondence or lasting correspondence.

Children at this stage identify collections arranged in such a way that it appears visually or globally to be approximately similar to the first set, using neither matching nor counting to quantify the cardinal number of the two sets. One example of such a response is that of a boy aged 4 years 4 months who counted out ten vases and laid thirteen flowers in front of the vases in a row of corresponding length. On finding three remaining flowers he removed them all as a bunch. When asked "Is there the same number of vases and flowers?" he replied No. When asked "Where are there more?" The child replied "There are more vases". A similar example is given by a teacher in Biggs (1970), who notes the children's 'intrigue' that the total of a collection of bottle tops should be the same as the number of bottles from which they had been previously removed.

Stage Two: One-to-one correspondence without lasting equivalence.

These children succeeded in arranging a second set which was equal in number to to the first. However, when one of the sets was subsequently re-arranged so that it appeared longer than the other they stated that the longer set was greater in number. The majority of children allocated to this stage were aged between 5 and 6 years. Piaget quotes a child aged 5 years 3 months who, on observing the equivalent set spread out in a longer row, stated that "There are more when it's bigger". As with many of Piaget's experiments, replication in recent years has yielded similar results; Dickson, Brown and Gibson (1984) attribute this error to the children's reliance on a 'perceptual property, in this case length, as their indication that a set contains a greater number of items.

Stage Three: Lasting Equivalence (Invariance)

In this stage the children are able to create a set of objects that is equal to the first set and are aware that they remain equivalent following rearrangement. In

Piaget's experiments the majority of the children were aged about 5 years 6 months. He cites an example of a child aged 6 who explained: "Yes, it's the same number of glasses. You've only put them close together, but it's still the same number" (Piaget 1952).

Piaget uses the results of his one-to-one matching and conservation tests to illustrate the difference between 'empirical knowledge' in young children and 'logico-mathematical' knowledge. In the above tests of one-to-one correspondence and conservation, most 5 and 6 year-old children are found to be able to identify the logico-mathematical relationship of one-to-one correspondence and can deduce or abstract from empirical facts that the researcher has one more bead in the task involving identical glasses for the adult and the child (Kamii 1994). It should be noted that these results apply to tasks involving collections of objects up to around 10 or 15 and may not apply to larger numbers for some time.

1.2 Research into early number post-Piaget

Research in this area of developmental psychology has expanded since the work of Piaget from the 1940's onward with follow-up studies by others which explore further aspects of the concept of cardinality more extensively, but some of which challenge Piaget's methodology or his interpretation of the findings.

1.2.1 Controversies associated with the design of Piaget's tests into the Logic of Number

Earlier research methodology, including that of Piaget, has recently been the subject of much criticism. Many have considered that some of young children's difficulties may be influenced more by 'noise in the system' than by innate inabilities or limitations, making tests less effective or valid in identifying the extent of children's understanding. This has resulted in a specialist field of research activity and debate within developmental psychology. It may be that Piaget and his colleagues assumed that their methods of interviewing children, using follow-up questions and approaching the same concept in different ways, were sufficient to ensure that the misconceptions they found were not simply a product of unfamiliarity with language or of being asked questions.

In an overview of this field of research Cohen and Cohen (1988) identify four variables in the young child's ability to be successful in tests of early number:

- The degree of the child's familiarity or unfamiliarity with the materials or conditions in the tests.
- The child's understanding of the language of the person carrying out the experiment or interview.
- The proximity between the child and the experimenter in both their personal interaction and in their shared understanding of the intentions and purposes of the tasks.
- The means of response available by which the child may express or convey his understanding.

Cowan (1991) summarises five possible reasons as to why younger children's responses differ from those of older children and adults:

- Differences in their understanding of the question
- What they mean by what they say
- The procedure used in order to arrive at their answer
- The results they obtain when applying a particular procedure
- The way in which they understand number (1991 p48)

These above aspects of research design highlighted by Cowan and Cohen & Cohen have themselves been the subject of recent debate and examination.

1.2.2 Contexts and materials used in more recent experimental designs

Piaget's tests in another cognitive field have sparked great interest in the contexts and materials used for testing young children generally and in early number in particular. These tests include that for egocentrism, or 'visual perspective taking' i.e. the ability to de-centre. The original test materials included a model of three different mountains and a doll and children were asked to select the picture of the mountain that the doll is able to see from where he is positioned among the mountains (Piaget & Inhelder, 1956). Most 6-7 year-olds, and many 8-9 year-olds, are not successful and choose the mountain which they themselves can see. More recently however such poor results have been partly attributed to using the context of mountains, rather than a setting more familiar to most young children.

Piaget's original test was later adapted by Hughes and the results served to draw greater attention to the importance of test design. Hughes (1975) replaced the materials by using dolls to represent a policeman and a boy, plus a model representing intersecting walls. Results of around 90% success with children aged only 3-5 years suggest that sensitivity to context and materials may well influence the results. Donaldson (1978) claims that as few children relate to mountains they view the task as psychologically abstract from 'basic human purposes and .. endeavours'. In the revised task Donaldson (1978) claims that children can make 'human sense' of the situation because the 'motives and intentions of the characters are entirely comprehensible', and it is the increased meaning to which she attributes their increased success.

Donaldson (1978) claims that very young children rely principally on the context as a source of meaning and understanding when problem-solving because they have had less experience, know less about language and as a result do not always know whether or not to take account of the context or take the researchers' questions literally. Cowan, Dowker, Christakis and Bailey (1996 p.86) consider that using unfamiliar situations or dialogue in number tests with young children might "unintentionally divert attention from counting principles" and "inadvertently challenge children's confidence in their counting" (Dowker & Cowan, 1998). Davis (1991) claims that when working with young children a meaningful context is necessary so that they can 'operationalize their skill'.

1.2.3 Children's interpretation of tasks and test questions

In his redesign of Piaget's Three Mountains egocentricity test Hughes (1975) considers the experimenter's language an important factor in enabling children to be more successful at a younger age than in the original tests. He had changed the overall structure of the task as well as the experimenter's use of language. The adult's questions were adapted so that the child's response was simplified to yes or no, whether or not the policeman could see the boy from each position. Secondly, the adult gave the child explanations and feedback on his first attempts. Thirdly the child's understanding was further explored by requiring him to hide the doll so he could not be seen by the policeman.

There has been further examination of the extent to which the child shares the experimenter's understanding of the purposes of the task. In Piaget's task for conservation of number, discussed earlier, Donaldson (1978) suggests that the adult's deliberate re-arrangement of items may pre-dispose the child to attach undue importance to the adult's action. Rose & Blank (1974) believe that when an adult repeats the question 'how many are there now?' it might cause young children to doubt that there could still be the same number even if they believed so. Rose & Blank draw attention to young children's sensitivity towards adult language when used in this way. Following tests of order-irrelevance carried out by Baroody (1984), Gelman, Meck and Merkin (1986), they too attribute children's poor performance to similar repetition. Davis (1991) highlights the significance for very young children of minor details in the wording of tests, such as repeating a question or not giving one piece of information.

It therefore appears that we cannot always expect young children to understand the questions asked in test situations, but further research shows that neither can we be sure that they will tell us when they do not really understand the task, or even refuse to answer when they do not understand. To examine whether children are prepared to query the wording of a test if they do not understand, Hughes and Grieve (1978) asked slightly older children (5-8 year-olds) questions that did not make sense, for example "Which is bigger, milk or water?" and "Which is heavier, red or yellow?" Although they could clearly not have understood the meaning of the questions, the younger children endeavoured to answer the questions and did not say that they had not understood.

Cowan (1991) highlights two points about the way younger children may receive language. Firstly, there may be differences in their interpretation of even such seemingly unambiguous words as 'same' or 'more'. Secondly, although it is possible to separate the meaning of a word from the way it may be interpreted in a given situation, 'in practice it is very difficult to disentangle them' (Cowan 1991). Donaldson (1978) suggests that the difficulty with testing is that there may be not so much a failure of the child to understand the concept or the task but a 'failure of communication' (Donaldson 1978 p44) and Davis (1991) emphasises the need to 'ensure that the child's interpretation of the question is as we had intended'.

1.2.4 The child's means of response

Empirical research designs include when and how subjects will be asked or enabled to respond to the task and the questions asked, however as described above we cannot always be sure that the child's response is an accurate reflection of his understanding (e.g. Siegel, 1978). In contrast to everyday settings, in test situations a child is responding to questions specifically formulated for the purpose of the adult obtaining a particular type of response. Donaldson (1978) draws attention to these 'linguistic settings' in which such tests often take place, observing that these may differ markedly from other situations in which the pre-school child usually finds herself. For example, as Bloom (1974), Donaldson (1978) and Slobin & Welsh (1973) point out, in the course of natural, 'everyday' conversation with an adult the child's speech is *spontaneous*: 'the child's attention is drawn to something that interests him and he speaks of it... and he expresses it in whatever form comes most readily to him' (Donaldson 1978, p.74). Slobin & Welsh (1973) found that even when a child was asked to repeat words that he had himself spontaneously produced earlier on, he became far less successful.

Bloom (1974) and Slobin & Welsh (1973) point out that when the child speaks on his own initiative he is not required to go against his 'own preferred reading of the situation ... the way in which (s)he spontaneously sees it' (in Donaldson, 1978). They comment that when the child speaks on her own initiative she is far more successful in her use of language, and suggest this is because the child's sense of 'intention..sustains and supports' more complex use of language (Slobin & Welsh, 1973). Slobin & Welsh (1973) suggest that when intention is absent, the child is required to use 'pure isolated language' and the task then becomes of 'a very different kind' (Donaldson, 1978).

Vygotsky (1962) suggests that this is because one's '..control of a function is the counterpart of one's consciousness of it' (in Donaldson, 1978). This may be due in part to the young child's cognitive limitations, as at this age she cannot easily reflect on her own use of language when in test situations. In this light Gelman & Gallistel (1978) consider that if children's difficulties arise from inappropriately designed tests, this is due to a 'failure of performance' and does not reflect a lack of 'competence' (Light 1986, p.184). There will always be an issue of distinguishing between when children's confusion reflects conceptual confusion or logical deficiency and when it merely reflects inadequate grasp of language of misconception of the experimental task. Clearly there cannot be a way of studying young children that guarantees that their responses are a reflection of what they understand of a concept. It would be easy to over-estimate what they know by using tasks that require them to do no more than they have already learnt, or by simply repeating what they have learnt to say. It is also possible to underestimate their competence by failing to adequately explain the task, or by making the task too demanding linguistically.

Davis (1991) highlights these difficulties thus: 'children's understanding of the test question and their interpretation of the task as a whole all add up to a more complicated picture of assessing knowledge'. She differentiates as follows between: '..children's knowledge and their understanding of when and how that knowledge should be deployed' considering that these 'are two separate problems' (Davis 1991, pp.44-45).

1.3 Tests for the logic of Number post-Piaget

Following findings on the issues involved in test design, discussed above, researchers in general have endeavoured to use more familiar contexts materials in tests involving young children, with close attention to the language used. Where Number tests require the child to respond orally there may be further 'probing', and the language the child uses in reply subjected to analysis and interpretation. They may also be designed so that the child points to or manipulate certain items as an additional indication of their understanding.

Current research thereby aims to explore not just the content and extent of young children's understanding, but also 'the features that are required for children at different stages of development to produce an appropriate understanding of the situation and the task' (Riley, Greeno, and Heller, 1983 p.192)

In the following account the original tests designs used by Piaget were changed in order to examine any difference in results.

1.3.1 Results of the re-design of tests for conservation and class inclusion

Conservation of number

Following reviews of Piaget's original design of the conservation test, it was suggested by Bryant (1974) and Samuel & Bryant (1984)) that when children are asked to compare the re-arranged second set of objects with the first set in Piaget's conservation tasks they need to understand and apply the principle of transitivity. They therefore claim that these experiments cannot accurately assess the children's grasp of the invariant nature of number. Where researchers have conducted Piaget's tests in an amended format, some results show a marked improvement in performance.

The conservation task was revised to allow for the difficulty caused by young children's sensitivity towards adult language highlighted by Rose & Blank (1974) and avoided repeating the question 'how many are there now?', with the result that children's responses subsequently improved. Bryant (1974) re-designed the task to make use of only a single row of objects instead of two, so children were required only to re-arrange the original row. In these tests 75% of four year-olds were able to indicate equivalence, compared to 35% in a replicated Piagetian test where a second set of objects was used as a comparison. McGarrigle & Donaldson (1974) also conducted an amended conservation test in which the second set came to be re-arranged not as the result of an adult's deliberate action, but as the result of an 'accident' following the clumsy moves of a 'naughty teddy'. Under this condition, over 60% of the children were able to conserve.

Miller (1984) found that pre-schoolers made use of one-to-one correspondence in the act of 'distributive counting' or sharing, and judged that this indicated an understanding of the principle. However, further work by Cowan (1987) demonstrated that these findings are only limited to small numbers, up to 6 or 7 objects being shared, with performance decreasing with larger numbers, i.e. 16 or 17 objects. Frydman & Bryant (1988) investigated whether children would make use of number to compare two sets that they know to be equivalent. Four-year-olds were asked to share out some 'sweets' equally between two puppets, and they did so using the 'one for you, one for me...' strategy. On completion the experimenter counted aloud the first set, then asked the children how many were in the second set. In this case the children did not make use of transitive inference, but began to count the second set aloud. Frydman & Bryant (1988) conclude that although able to count, four year-olds do not recognise counting as an indication of set size.

Class inclusion

In the original class inclusion task of Piaget & Inhelder (1967), in which children were asked: 'are there more tulips or more flowers?', most four-year-olds answered: 'more tulips' and 'than roses' which led Piaget to conclude that young children are not able to think about and compare two sub-sets with the whole set until around the age of 7 or 8. The task was later adapted by McGarrigle (McGarrigle, Grieve and Hughes, 1978) to involve a Teddy, a small table, small chair, and a set of four plastic counters representing stepping-stones along which Teddy travels to reach the chair or the table. In this case, the questions asked were: Are there more steps to go to the chair or more to go to the table?' In this re-designed version McGarrigle found that over two-thirds of children aged 3 to 5 years were successful.

It is important to note that where Piaget's original designs are used, the results remain the same, raising issues as to whether this reflects children's difficulties with the way the tasks are designed or the constant nature of young children's inability to grasp the logic of number.

1.4 Early quantitative understanding and counting

Current research explores not only the content and extent of young children understanding, but also the means by which young children learn about early number. Riley, Greeno, and Heller (1983) define this as involving:

i) an analysis of the process of young children's understanding in various problem situations

ii) an account of the features that are required for children at different stages of development to produce an appropriate understanding of the situation and the task (Riley et al 1983, p.192)

Tests carried out more recently also aim to identify the processes by which children develop concepts. Such research takes a broader view and explores the early origins of number awareness in babies as well as investigating subsequent progression and the skill of counting in young children.

1.4.1 Infants' early number skills

In studies of cognition an awareness of number has been observed in babies of less than a year old. These studies report a recognition of difference in set size where sets contain up to three objects (Starkey, Spelke and Gelman, 1990). Wynn (1992) reports that babies as young as five months react when objects are removed from or added to a set and the set size changes. This recognition is seen by some to be the result of 'subitizing', a visual sweep of the set as a whole. Infants are found to respond in this way with sets of up to three objects, which is seen as the 'subitizing range'. Children as young as two years of age were found by Starkey and Cooper (1995) to be capable of subitizing objects in a variety of arrangements. Klahr and Wallace (1976) suggest that subitizing is a process of quantification that develops prior to counting.

Wagner and Walters (1982) also attribute infants' number discrimination of up to 3 objects to visual perception, but claim that what infants take account of is the space between the objects. With one object there is no space and the single object is a continuous solid item. In contrast two objects are identified by having a continuous space between them. Three objects may be identified by discontinuous space whether objects are presented in a linear or non-linear arrangement.

Canfield and Smith (1993) presented a series of slides to infants to their left or right hand side in a variety of sequences. They found that babies of 5 months formed expectations about the numerosity in the pictures shown; Canfield and Smith (1993) believe that their responses are based on the counting principles identified by Gelman and Gallistel (1978), using 'internal count tags' instead of words. Gallistel and Gelman (1992) also attribute infants' recognition of small numbers of objects to a form of non-verbal counting based on 'accumulation'. This is similar to Resnick's (1992) 'protoquantitative reasoning' which identifies the effect of an increase or decrease on unmeasured quantities of objects.

As Sophian (1995) suggests, these findings serve to highlight the need to consider that infants may not be 'carving up the world' as we do. There is however relatively less information at present about any relationship between these early responses and later development of counting and the concept of number.

1.4.2 Recent research into how counting develops

In domestic and pre-school settings the term 'counting' is often interpreted initially as the ability of the young child to verbalise aloud the string of natural numbers. Once this has been mastered there follows an expectation that the child will learn how to select one or more objects and to count or quantify them; it is at this point that the skill of counting *objects* is recognised as requiring considerable advances in the child's conceptual development, making this both a difficult and usually more lengthy skill to master.

Piaget's belief that the limited nature of pre-school children's logico-mathematical understanding led him to view counting at this age as 'just verbalisation' (Piaget 1953) and a rote procedure not connected to an understanding of quantity (Dowker & Cowan, 1998). His experiments have since been widely replicated and current research is examining the sequence in which young children develop associated conceptual structures alongside counting procedures.

1.4.2.1 Gelman and Gallistel

Gelman and Gallistel (1978) identify three initial principles that the child first needs to learn and apply when counting objects successfully.

i) The first principle is the rule of the 'stable order' of number-names or number-tags, in which these need to be used in the same order at each stage in the process and cannot be changed. (This is dependant upon success at the earlier stage of simply reciting the sequence of number names, in whatever form or language they may be.)

ii) A second principle is that of 'one-to-one correspondence'; this states that each item is indicated and counted once only, and each assigned only one number 'tag' or name. [This involves remembering which objects have already been counted and which are still to be counted.]

iii) Having set out and noted each object, the child is then required to demonstrate the principle that is third in this hierarchy, that of the 'cardinal' use of number. This rule states that the last number assigned to an object represents all the members of the set previously counted and is used to specify the quantity or size of that group of objects. Implicit here is the principle of cardinality, that the same cardinal number is reached no matter how many times the same set is counted and in whatever order.

As shown in Piaget's experiments, the process of assigning a number to an object belonging to a sequence is the 'ordinal' aspect of number, and the step of knowing that the final number can be used to represent the numerosity of the set as the 'cardinal' aspect. This is the link between the ordinal and cardinal aspects of number. Gelman & Gallistel further indicate that once the three principles of stable-order, one-to-one correspondence and cardinality are applied, two further developments are necessary for a secure grasp of the counting procedure.

iv) The principle of 'order-irrelevance' - that the various count-words can be applied to any object and starting with any object - so long as each count-word is used only once during the count. This indicates an understanding that any particular count-word is arbitrary and does not belong to any particular object, (eg. 'three' to a train in a line of a toys); on the basis of this principle the train may be counted first or even last when the set of toys is counted again.

1.4.2.2 Fuson

Fuson (1988) identifies five levels of complexity when learning to count objects successfully. At first the child achieves initial mastery of the sequence by reciting an unbreakable, uni-directional string of numbers. This includes a stage in which children acquire the flexibility to continue or reverse this sequence at any point. She also identifies the ability to identify relationships between different numbers.

Fuson (1988) is among others to note that the earlier studies by Gelman and Gallistel involved children counting sets of objects that were pre-arranged in straight lines with regular spacing Fuson considers that this arrangement may not have challenged the depth of the children's understanding sufficiently stringently. She claims that their use of the counting procedure may have been an automatic response to the ordered arrangement.

Follow-up research by Fuson (1988) required children aged between 3 and a half to 6 years, to consider several collections of objects which increased in size from between 4 and 40 in number. She also asked them to count dots fixed onto card in the form of circle so that they could not be moved. Her results show that when the objects are re-arranged and spread out randomly, all children have greater difficulty in adhering to the principle of one-to-one correspondence.

Fuson's results did reveal some improvement with age. Older children, aged 5 and a half to 6 years, spontaneously moved the objects they had already counted to one side, achieving the same level of success as when the objects were set out in a row. However this was not reflected with the fixed dots as only one of the 16 older children attempted to mark the point where counting had started.

Piaget's tenet that children's mathematical understanding (in this case counting) reflects their grasp of the principles of logic is explored by Fuson who raises two considerations:

i) The first hypothesis is that the principle of one-to-one correspondence has been established and the error may simply be due to the children losing track of which items had already been counted.

ii) The second hypothesis is that the principle is not sound and the children may be counting automatically, guided by the regular layout and spacing of the objects along the row.

1.4.2.3 Munn

Work on counting by Wynn (1990) with American children, which identified a move in their thinking from 'grabbing' an estimated amount to 'deliberate' counting at around the age of 3 years 6 months, was followed up by Munn (1994). Forty-six British children were tested on their skill and understanding of counting at the beginning and end of a nursery school year. Mean ages were 3:10 at the start of the study and 4:7 (of school age) at the end of the study. The children were firstly asked to give specific numbers of bricks, and asked questions about where and how they had learned to count.

By the end of the year, 31 counted spontaneously when asked to give quantities of bricks, 15 judged quantities by eye even though they knew how to count, and 23 used numerals to represent quantity. Considerable changes were noted across the year, with many 'grabbers' moving towards counting as they neared school age. Munn (1994) attributes this as due less to improvements in counting skill as to their 'strategic application' of this ability between the ages of 3 years 10 months to 4 years 7, i.e the ability to use and apply the strategies.

Munn (1994) suggests that the 15 who judged quantities by eye, in spite of being able to count, did not consider "Give me X" to mean the same as being asked "How many?" and did not see counting as 'relevant to the situation'. This endorses Bryant's view (1997) that young children who are able to count still do not know when and how to make use of number to compare the sizes of sets, and Sophian's work (1988) which showed that 3 year-olds did not appear to realise that it is necessary to make use of their learned counting procedure.

1.4.2.4 Cowan

Cowan (1987) examines the view that counting has itself a major role to play in young children's development of the number concept. He considers explanations for research findings which show that young children who know how to count nevertheless make judgements about relative number which are inconsistent with their apparent counting skill. Cowan summarises these possible explanations:

i) A lack of counting proficiency. [Although Cowan considers this to be unlikely in the case of three year-olds in the light of research by Gelman & Meck (1983)].

ii) Not knowing how to adapt counting to identify the cardinal number, as according to Durkin, Shire, Riem, Crowther and Rutter (1986b) most children have experience in counting *one* set of objects.

iii) Not understanding 'relative magnitude' when comparing two unequal quantities, i.e. that one needs to record, remember or 'store' the result of undertaking counting each set. However, as Cowan points out, research in this area (Schaeffer, Eggleston and Scott, 1974; Siegler & Robinson, 1982) did not make use of visible objects, and did not examine the relationship between knowledge of digit size to the children's counting ability.

iv) Forgetting the number of items included in both or one of the collections of objects.

Factors associated with the design of the tests are also considered by Cowan (1987) as having possibly influenced the children's success in making relative number judgements. Brainerd (1977) for example examined the layout of three parallel rows of objects, finding that 5 to 7 year-olds judged displays in which the two rows were unequal in number but equal in length to be right more often. When the displays were set out as unequal in number and unequal in length, the children were more commonly wrong in their judgements.

In follow-up studies, Cowan obtained similar findings according to the layout of the display (1987a, 1987b), despite in the second study having selected children who were competent in counting (1987b). Cowan also found that providing a set of physical guide lines for indicating one-to-one correspondence between the two sets considerably improved the relative number judgements of the 7 year-olds, but not those of the 5 year-olds.

A later comparison (Cowan, Foster and Al-Zubaidi, 1993) with children aged between 5 and a half and 7 and a half years in the Yemen again selected children already able to count. Four different conditions applied, with two experimental and two control groups. In the two experimental conditions, children received verification that counting is effective in comparing sets. In one of these groups they were shown one-to-one correspondence, counting, and the fact that both yielded the same result. In the control conditions the children received the same amount of experience in judging the relative number displays and were encouraged to count, however they were not given any verification or feedback on their performance.

Although there was an all-round increase in performance only the experimental groups showed a marked improvement. Just over half of the Yemen non-counters used counting after training, compared to 32% of the control groups, with 6 and 7 year-olds showing the biggest increase. A second part of the study worked with children in London. These results differed slightly, with just under half of the younger children counting on every trial. Cowan's results (Cowan et al, 1993) show that whereas young children may be able to count a set of objects correctly and give the cardinal number, they may not yet have an appreciation of size or quantity of the set. Even when directly encouraged to count they did not make a comparison or relative number judgement. Cowan points out that we need to be aware of the 'fragility of children's trust in counting' (Cowan, 1987a).

1.4.2.5 Wynn

Wynn (1990) describes the principles defined by Gelman & Gallistel (1978) as "innate how to count principles .. which exist before children have any experience of counting" (pp.156-7). In a later study by Wynn 2 and 3 year-olds were asked "how many objects?" following a count of up to six items, and then asked to "give *n* objects" to a puppet.

Although able to 'give' one and two objects correctly, Wynn found that a significant majority of the younger children did not use counting but 'grabbing'. In contrast when asked 'how many' objects there were, success was significantly related to age with a majority of the older children using counting to provide the correct numerosity.

Wynn concludes that children do not start with a set of (innate) principles concerning the importance of counting which guides counting 'behaviour'.

However Wynn agrees with the findings of Gelman & Gallistel (1978) and Gelman et al (1986) that young children do not grasp the cardinal word principle until the age of around 3 and a half years. She describes the development of children's understanding of counting as 'complex and piecemeal'. For example, she suggests that very young children may have an innate concept of numerosity, seemingly for small quantities, which they later 'map' onto the correct number words of one, two and three. She suggests that they may interpret the last number word as 'the answer', when questioned about how many, rather than as providing the cardinal number of the set. Once children grasp that the last number word represents the numerosity of a set, Wynn suggests that they then use the knowledge to learn the meanings of all the number words in their current counting range.

1.4.3.6 Sophian

Others such as Dehaene (1997) and Sophian (1998) propose that it could be that infants' innate biological structures provide a predisposition and sensitivity towards number - as in the accumulator model used even by young babies. Sophian suggests that the accumulator mechanism itself 'embodies' the counting principles discussed by Gelman & Gallistel (1978) and that it is these principles which subsequently focus the child's attention on the same key features in the counting process.

Sophian goes on to give consideration to the child's own biological goals and the adult goals inherent in joint social activities (such as counting). She points out that as young children participate in these activities their attention is directed to certain aspects of number use from which particular conceptual development may occur and which in turn prompt further conceptual advances (p.46).

1.4.3 The relationship between conceptual, procedural and utilisational competences in counting

Further research into early number examines the relationship between early conceptual, procedural and utilisational competences. Rittle-Johnson & Siegler (1998) define conceptual development as an 'understanding of .. goals and principles', often referred to as 'principled knowledge'. They define procedural knowledge as "action sequences for solving problems - sometimes referred to as skills, algorithms or strategies".

Gelman & Gallistel (1978) conclude that young children's counting is preceded and underpinned by a knowledge of principles as these could not have been learned by imitation and as even those children whose counting procedure is not totally correct follow these principles.

Some have argued (e.g. Baroody, 1984) that variability in young children's performances in counting demonstrates that they do not have a grasp of underlying principles. Some consider that a grasp of underlying principles guides learning to count (e.g. Gelman & Meck, 1983), whereas others see children's experience of counting *leading to* the development of principles (Siegler, 1991).

1.4.3.1 Gelman & Meck

Gelman & Meck (1986) propose an analysis of how children's principles might be translated into practice. Gelman et al (1986) distinguish between conceptual, procedural and utilisational competence, and their suggestion is that this variability in counting performances is due to two factors. Firstly it may be due to problems that children experience in assessing the requirements of the task - problems known as utilisational competence, i.e. knowing when it is relevant to apply their conceptual knowledge. Secondly, it may be due to problems in planning the solutions, i.e. procedural competence.

Gelman, Meck and Merkin (1986) claim that while the young child begins with 'some implicit understanding of counting' which guides the acquisition of skill (p28), the skill or procedural competence then 'begets' further understanding. Conceptual competence is therefore viewed by Gelman et al (1986) as a pre-requisite for the development of both procedural, and further conceptual, competences.

1.4.3.2 Bryant

Bryant (1997) questions whether the way in which children undertake counting, sharing and adding at ages 3 and 4 is truly mathematical. He draws attention to research findings that young children who are able to count still do not know when and how to make use of number to compare the sizes of two different sets. He refers to the earlier work of Sophian (1988) which showed that when 3 year-olds watched a puppet comparing the size of two sets of objects they did not appear to realise that it is necessary to make use of counting. He also refers to the findings of Cowan & Daniels (1989) that even when visual guidelines are introduced to encourage the use of one-to-one correspondence, they do not do so. In other words: "children often count without understanding counting .. and may well imitate" (Bryant 1997, p.61). In line with the view of Piaget (1952) and Fuson (1988), Bryant suggests that "at first children are practising little more than a verbal routine when they count" (p.60).

In follow-up tests Bryant (1997) examines whether or not young children's apparent proficiency in the skill of simple sharing (Miller, 1984; Frydman & Bryant, 1988) is underpinned by an understanding of why this is the right thing to do, or the result of imitation and verbal routine. A group of 4 and 5 year-olds were asked to share out fairly a set of 'chocolates' between two recipients; some recipients would only accept 'doubles' (chocolates arranged in fixed pairs) and others only singles. Bryant found that only the 5 year-olds were able to account for the double amount by adapting the usual routine of "one for you, one for you".

In a later task each double chocolate was given in two colours, and each pair of singles was also given in two (different) colours. Bryant (1997) suggests that this alternative arrangement has the effect of emphasising one-to-one correspondence, although he also believes that children of this age do have a grasp of the reason why one-to-one sharing leads to equal quantities for both recipients, and that they do have a basic understanding of cardinal number. However, in further tests, when told the amount of sweets given to one recipient, none of the 4 year-olds made the inference that the other must have the same amount again casting doubt on their understanding.

1.4.3.3 Rittle-Johnson & Siegler

A review of the relationship between children's understanding of mathematical concepts and their ability to carry out procedures which embody these concepts is carried out by Rittle-Johnson & Siegler (1998).

Their premise is that conceptual and procedural knowledge "without question develop in tandem rather than independently". They see this tangential development as "iterative..a hand over hand interaction..incremental" (Rittle-Johnson & Siegler, 1998 p3).

Rittle-Johnson & Siegler (1998) set out a review of research which shows how forms of knowledge 'interact', a process which they consider to be the basis of 'how development occurs'. They relate their findings to the hypothesis of 'privileged domains' (Geary, 1995; Gelman, 1993) in which a few mathematical competencies are identified as being easy to learn due to 'potential evolutionary importance'. These domains are identified as competencies which are universally developed at a young age, the most commonly identified being the procedural skill of counting, and simple arithmetic. In particular, children may frequently see counting in use and participate (termed 'frequency of exposure'), which allows them to 'abstract its conceptual underpinnings' before a later conceptual understanding of the underlying principles.

In these instances of 'privileged domains' early conceptual development is said to guide the acquisition of procedures, and these domains then develop more rapidly than others. In contrast, the 'non-privileged' domains are identified as those in which both concepts and procedures develop more slowly. However, due to the few cases in which they find that conceptual knowledge is developed but not used to solve problems, Rittle-Johnson and Siegler (1998) also hold some reservations and recommend that:

- the 'frequency of exposure' hypothesis is considered in greater detail
- assessment needs to be refined in order to record gradual changes in the often protracted period of progress in younger children, and to be more stringently matched to similar tests of older children in order that valid comparisons can be made.

1.4.3.4 Dowker and Cowan

Dowker and Cowan (1998) suggest that from the viewpoint that principles guide the ability to count:

a) children who are asked to count the same set at different times and in different orders would expect the same cardinal value

b) this could support the development of accurate counting by serving as a 'basis for monitoring counting'. According to Dowker and Cowan (1998), if one takes the 'skills first' viewpoint it is only when children's counting skill is sufficiently reliable and accurate to provide the same cardinal value after counting the same set in different orders that they discover that order is irrelevant.

Methods of investigating children's grasp of order-irrelevance have varied, as have the findings, and Dowker and Cowan (1998) suggest that the variation in findings relates partly to differing tasks. These include those in which the child may not have viewed order as irrelevant but being simply 'indifferent'. Gelman et al (1986) found most 3 and 4 year olds were successful in a test of order irrelevance which did not require them to count.

Dowker and Cowan (1998) aimed to identify the causes of varying test results, and whether the children's attention to underlying principles is interfered with when asked to count. Experiments were designed to examine the empirical relationship between counting skill and understanding of the counting principle of order-irrelevance (Dowker & Cowan 1998).

Indifference to counting order was investigated by two tests. 'Constrained counting' required children to recount a set with one item having a specified number (Gelman & Gallistel, 1978; Gelman & Meck, 1986). The tests required the child only to predict the result of recounting a set of objects; the number of objects was kept small as in tests with larger numbers the action of recounting "may challenge confidence considerably more than a smaller numerosity" (Dowker & Cowan, 1998).

These young children varied in their grasp of order-irrelevance and in their understanding that adding or subtracting changes the number. Dowker & Cowan (1998) note that the principles were applied firstly to small numbers & only later to larger numbers (p.5). Performance was influenced particularly by counting ability, however some accurate counters understood order-irrelevance and others did not.

Dowker & Cowan (1998) identify two points from these findings:

a) proficient counting was linked to more success with small numbers;

b) all children were more successful with the subtraction task, regardless of counting ability.

They suggest that "order-irrelevance may first be applied to small numbers and only subsequently generalised to larger numerosities" (Dowker & Cowan 1998, p.5). This is consistent with findings related to the development of other principles of early number such as conservation and judging relative number or one-to-one correspondence (e.g. Cowan 1987b; Gelman 1982). Dowker & Cowan suggest that this "calls into question the 'skills first' versus 'principles first' dichotomy" (1998, p.5) and leads Cowan to stress the need to be aware of the 'fragility of children trust in counting' (Cowan, 1987a).

1.4.4 Counting and its relationship with later development of the number concept and numerical thinking

Whereas many studies have examined the process and rate of children's ability to count, only recently has counting been considered in relation to subsequent development in other areas of early number. Bryant (1997) in particular questions whether there is a connection between these counting and sharing activities at age 3 and 4 years and later progress in mathematics.

In tests by Dowker & Cowan (1998) young children varied in their grasp of order-irrelevance and in their understanding that adding or subtracting changes the number. They note that the principles were applied firstly to small numbers and later to larger numbers (Dowker &

Cowan, 1998, p.5) They propose that an alternative account may be that of Baroody (1992) and Rittle-Johnson & Siegler (1998) who suggest that the process may be 'interactive' in that counting skills and conceptual knowledge could 'mutually' support development in the other aspect i.e. that counting may well play an important part in developing an understanding of number.

Resnick (1992, in Sophian, 1996) suggests that the development of numerical thinking is a lengthy process, identifying four different levels, in which counting plays an important part:

i) Protoquantitative - non-numerical (i.e. unquantified) schemas of babies and infants for understanding the effects of actions which result in increasing or decreasing a given quantity.

ii) Quantitative reasoning - reasoning about measured quantities of objects once they grasp the number system, including one-to-one correspondence, conservation and, later, many-to-one correspondence.

iii) A mathematics of numbers - when numbers themselves are considered, i.e. as the object of thinking and reasoning; this stage is likely to be influenced by the process of instruction and formal representation.

iv) A mathematics of operators - focusing on the relationship between various mathematical operations, as taught in schools from the middle Primary years and later including algebra.

Late twentieth century research then pays close attention to the period in which very young children develop an interest in number and subsequent conceptual and procedural knowledge, and highlights the complex nature of this aspect of children's development. In line with Cowan's recommendation for a 'broad and comprehensive' approach the following section will examine recent theories and research on the means by which the young child's mathematical development is fostered.

1.5 Environmental factors involved in learning about number

In any discussion on counting we are constantly reminded by the work of Piaget (1952) that in his view learning to count does not in itself constitute learning to understand number for two reasons: a) learning to count is not sufficient to bring about changes in understanding of number e.g. conservation; b) children may develop an understanding of number independently of learning to count.

Cowan et al (1993) deduce that we may have placed too great an emphasis on studying the child as an individual, rather than looking at the support available within the society or culture. In reviewing findings that young children fail to link counting and cardinality for some time, Fluck (1995) raises the issue of whether this may be attributed to either the child's universal stage of development, as in the Piagetian model, or to an absence or limited support within the culture. This contemporary stance to research emphasis addresses 'environmental' influences in relation to children's development of the number concept, rather than biological as did much of Piaget's work (e.g. Rittle-Johnson & Siegler, 1998, p.26).

1.5.1 The nature of socio-cultural support for counting

As we have seen, Piaget's work emphasized innate influences, biological structures, stages of cognitive development and language as the result or outcome of cognitive processes. Although Piaget refers to the child's interaction with the world around him, there is little attention paid to the role of adults in the child's development, the part played by the language heard by the child or the language used by the child as 'inner speech' (Vygotsky, 1962) for 'self direction' etc. (Tough, 1977). Following recent advances in constructivist research, which recognises the active part played by the child, the environment and society in young children's cognitive and linguistic development, attention has eventually fallen on the mathematical domain of the young child's development.

External or environmental support in the home (the 'socio-cultural' context) for children's counting was investigated by Saxe, Guberman and Gearhart (1987). They examined the nature of the support given by mothers of children in working class and middle class families in New York at the initial level of reciting the number sequence. They found that individual adults differed in the quantity of number-words they expect of a child at any given age, and that learning of the number-sequence continues to develop up to or beyond ages 7 or 8. The level of support offered by mothers for their children's number activities was also found to differ according to class, with lower levels of complexity and personal involvement found among working class families.

In a study similar to that of Saxe et al (1987), Fluck (1995) investigated the nature and extent of support provided by mothers for their children aged between 2:7 and 3:4 with regard to the development of counting at the level of cardinality. In Fluck's study (1995) the mothers were instructed to ask their children to count sets of objects for a clown doll, while he observed the mothers' introduction and responses. Fluck (1995) found that generally the mothers matched their responses sensitively to their children's levels of numerical development, but with many individual differences between mothers.

Fluck found that when following up the child's count, the mothers used the terms 'how many?' and 'count' interchangeably, and in only very few instances did they explicitly ask 'how many?', an action which might prompt a reply giving the cardinal number. Where mothers *did* ask 'how many?', the children replied with either the next word in the number sequence, or an arbitrary number (possibly an estimate) rather than the cardinal number.

After the child had counted, 62% of the mothers repeated the last number word, but on only *'some'* occasions, e.g. "one two three ...*three!*". The repetition came either with or a plural noun to make reference to the items counted, saying for example: "*three* cats", or without a noun, "three" with no reference to the items counted. Durkin (1993), in a similar study, noted that in most cases the mothers used a noun only on occasions when the child had counted correctly.

In Fluck's study (1995) eleven of the thirteen children gave a numerical type of response when asked 'how many', but they did not repeat the last or cardinal number. Fluck suggests that although none of the children understood the relationship between counting and cardinality, they did show a 'growing distinction' of the last word, and were able to distinguish between the meanings of 'count' and 'how many'.

1.5.2 The effectiveness of socio-cultural support for counting

A child's degree of success in reciting the counting sequence appears to be related to the language the child speaks and in which the numbers are learned; some modern languages have a pattern of number-words which renders the sequence more predictable and therefore easier for the child to learn (Fuson, Richards and Briars, 1982; Siegler & Robinson, 1982), particularly with numbers above 10.

Durkin (1993) suggests that repeating or vocally emphasising the last number of a count may be the way that caregivers provide support for understanding cardinality, i.e. in an implicit way, rather than providing a more explicit and instructional form of support. Fluck (1995) describes this 'linguistic support' (or 'scaffolding', Bruner, 1977) as the way in which caregivers provide the help needed by children for the eventual 'discovery of the connection between counting and cardinality' (p.147).

Another facet of this research with very young children examines the relationship between the nature of caregivers' support and their children's successful grasp of counting and cardinality. In the study carried out by Saxe et al (1987), it was found that lower levels of involvement and complexity were associated with lower levels of children's mathematical understanding and competence. They also found that activities of an 'everyday' nature actively enhanced children's mathematical development and that success was related to the frequency of teaching and of opportunities for the child to practise saying numbers aloud.

Further findings by Fluck (1995) reveal that while the mothers repeated the last word of the child's count on only *'some'* occasions, the children also vocally emphasised the last number word on only *some* of the sequences (23%). Fluck (1995) concluded that the mothers were responding as though the children understood the purpose of counting (cardinality), and this was a view that the mothers endorsed when interviewed.

Markman (1989) found that young children focus solely on the object being counted and do not grasp the function and meaning of counting or attend to the 'perceptual attributes' of the objects, such as type, or the 'thematic relations' between objects. Markman considers whether this is because children are seeking to reduce the difficulty of gaining meaning from the new count words, however Fluck

(1995) observed that mothers repeated the last number word on only *'some'* occasions either with a plural noun (e.g. "*three* balls") or without a noun, ("three"). Durkin (1993) notes that most mothers use a noun only when the child has counted correctly, while assigning the greatest weight to the count sequence itself through "pointing..and uttering a word" (Durkin 1993, p.146) which may well divert children's attention away from the objects or their perceptual attributes.

Durkin considers that early difficulties in number, and problems which develop later, originate from this limited knowledge basis. In his findings that many young children make judgements inconsistent with their counting skills, Cowan (1987) highlights the significance of the socio-cultural process in learning to count and in children's all-round number development. Rittle-Johnson & Siegler (1998) similarly suggest that support within the child's immediate culture makes a large impact on young children's number development.

On the face of it, the range and differences in children's performance found in the studies carried out by Saxe et al (1987), Durkin (1993) and Fluck (1995) appear to directly reflect the range and differences in the support for counting and cardinality provided by caregivers in the home. However further, controlled tests would be needed to explore correlations between children's performance and the specific nature of caregivers' support for number.

1.5.3 Models of socio-cultural support for counting

As the above studies indicate, one branch of developmental research in recent years has begun to examine in some detail not only the incidence of support for number during infancy and the pre-school period, but also the extent and nature of support provided by the environment.

While the support provided by the mothers in his study is identified by Fluck (1995) as 'incidental' rather than intentionally 'pedagogic', it is considered to be similar to the support provided for children in other cognitive domains (Bruner 1983; Rogoff 1990). Munn (1997) sees such support for counting as taking place on two levels. Firstly, on the 'social plane' in joint activities when children's learning is supported by 'adult language and goals'; similarly, Durkin (1993) refers to the 'cultural practice' in which caregivers involve children in joint activities. Munn sees this process as becoming internalised at a later stage, taking place on the 'psychological plane' and according to adult principles.

Dowker & Cowan (1998) propose that an alternative to the 'social then psychological' model suggested by Munn (1994) is that first proposed by Baroody & Ginsburg (1986) and later endorsed by Rittle-Johnson & Siegler (1998), who suggest that improvements in counting skills or conceptual knowledge could mutually support development in the other aspect through interactive mutual development.

Munn also discusses adults' support for counting (Munn 1997, pp.17-18) and raises the issue of 'whether practice leads to principles or vice versa', also discussed by Nunes & Bryant (1996). In their review of research findings across five different mathematical domains, Rittle-Johnson and Siegler (1998) generally support the view that conceptual and procedural knowledge are positively correlated and that extensive experience is related to enhanced development. They suggest that this relationship is related to the large impact made by socio-cultural or environmental support rather than to biological differences (p.26).

However Rittle-Johnson and Siegler (1998) set out the need for further research with:

i) more detailed task analysis which identifies multi-faceted conceptual development;

ii) repeated assessment of knowledge during the learning process;

iii) true experimental designs that can provide causal evidence of relations between concepts and procedures.

1.5.4 The relationship between socio-cultural support and the meanings that young children attribute to number

In the move away from a behaviourist approach developmental research now takes more account of children's individual thought processes, intention and meanings. Studies now seek to access not only children's understanding but also the sense that they make of the situations and questions they encounter when learning mathematics.

Light & Perret-Clermont (1989) identify three strands of interest in research into how young children learn about number:

i) interpersonal and discursive cues

ii) the 'social mechanisms' of cognitive change

iii) the 'pragmatic, inter-subjective agreements-in-meaning' identified in young children's cognitive development.

Solomon (1989) takes the view that learning about number is a question of "knowing how and when to use numbers", and that the *meaning* and *use* of number words are closely interrelated in the first few years. Hiebert (1988) refers to the process of "connecting individual symbols to referents of quantity", suggesting that as children cross 'bridges' in between the two, specific meanings are created for number.

This contemporary line of discussion deduces that the nature of the meaning which children construct for number and counting is also related to the quality of the support provided by adults, particularly in the home. Munn (1994) carried out research with 46 children in a Scottish pre-school during the year before school entry in order to map their personal meanings about number to their ability in this area (p.11). Most of the children believed they had taught themselves to count i.e. they could not recall being taught but saw counting as a 'linguistic activity', e.g. reciting numbers, rather than as a meaningful activity that is useful for discovering quantity. This is when numbers are heard in the context of a word string, e.g. "one step, two step..".

Fluck's research (1995) with children aged between 2:7 and 3:4 showed that although none understood the relationship between counting and cardinality they did distinguish between the meanings of the terms 'count' and 'how many', indicating a growing distinction of the importance of the last word. However Fluck considers it likely that the children were attributing the last number word with the meaning of 'task completion' rather than cardinality, as does Wynn (1990).

Sinclair (1991) concludes from her data and that of Frydman & Bryant (1988) that it is young children's everyday experiences with 'written numerals' that results in "the lack of salience of cardinality " (p.64). Munn (1994) similarly suggests that whereas young children share the adult's purpose in reading this does not apply to learning about number. She deduces from her findings that counting plays a very different role in children's lives than in adult lives, differing "markedly in function" and appearing 'purposeless' to the child (pp.16-17). She suggests that it is not until a later stage that children come to share "adults' belief systems about these everyday (counting) activities".

Durkin (1993) identifies two sources of difficulty in early number for young children:

i) confusion as to the meanings of the words used by adults

ii) the nature of the context in which a problem is presented

Durkin (1993) makes the point that: "Children do not appear to form concepts of numbers then map number words *upon* them" (p.148) as learning about number involves *a social process* when children have to discover what other people *'mean'.* Durkin highlights the meanings which adults share during their interactions with young children, and concludes that this calls into question whether it is innate factors which are largely responsible for young children's limited use of cardinality.

1.5.5 Comparisons with research into socio-cultural support for learning about text or the written word

Munn has compared and contrasted research findings about how children learn about counting and number with research into children's learning about books and reading. In relation to reading, Munn (1994) discusses the child's exposure to an "entire cognitive environment" with interactions in which adults convey facts about language and reading which helps children to form a metacognitive framework about reading and text. In the case of reading children are found to hear the 'language of print' when texts are read aloud to them; Goodman (1980) and Wells (1981) for example describe how children are often involved in interactions with adverts or in adult activities and conversations in the home, most of which are on a one-to-one basis. Findings show that such interactions prepare children well for learning to read.

Munn (1994) suggests that numeracy skills develop "in a similar way to language skills themselves". She discusses how number-related activities help provide a 'framework' which develops an understanding of basic quantitative tasks when "much...is learnt informally...about the role...of counting" (p.7). Munn gives the example of the activity of matching number words to actual quantities during number rhymes as producing an "overall mental set towards...number".

However Munn (1994) found that whereas children share the adult's purposes and meanings in reading from the earliest stages, counting "had no such stable framework which was shared with adults". For example, as Durkin (1993) describes, before school children may not normally undertake counting in order to know how many objects there are, but do so only when they are required to by an adult.

Munn (1994) concludes that children eventually come to share the belief systems of adults either "..explicitly through explanation, or implicitly through joint action" (p.16).

1.6 Summary

A considerable amount of research has already been carried out into children's understanding of counting and the logic of number. Piaget's view that logico-mathematical development takes several years to attain is a view which has since had an important influence on teachers' expectations of children in and beyond the nursery and infant school. More recent research has investigated the support available for learning in these domains within the culture including the home and school.

According to Sophian (1996) three main theories of cognitive development currently influence the study of young children's number development.

i) Piaget's focus on the 'pure' logic which he sees as underlying mathematics, his theory of 'logico-mathematical' development based on his view that innate structures result in the logical organisation of children's thinking, rather than children's grasp of 'cultural' practices such as problem-solving.

ii) Domain-specificity: separate areas of cognition which have their own specific biological principles of organisation, of which Number is seen as one such specific domain.

iii) Socio-cultural theories based on the work of Vygotsky, i.e. theories which focus on the influence which practice and interaction with the environment or culture may have on what and how children learn about number.

iv) The contemporary view of number, such as that of Durkin (1993) and Wagner & Walters (1982), as a multi-determined social activity in which all three of the above influences are seen to interact with each other.

v) The current discussion that logic, culture and teaching interact.

Linked to these theories is the use of empirical tests to gauge young children's understanding and interpretation.

Nunes' view is that each of these aspects 'need to be unravelled if we are to understand' (Nunes, 1997, p.48), and that this is particularly important where cultural practices are found to 'sometimes help and at other times actually hinder' children's understanding - including the use of certain mathematical contexts and symbols.

In line with the suggestion made by Cowan et al (1993) that we may have placed too great an emphasis on studying the child as an individual rather than looking at socio-cultural support, one branch of research aims to understand children's meanings and how children eventually come to share the belief systems of adults. This includes work on comparing studies in the number domain with studies of how children learn to read and write.

Uses of early number other than cardinal have been relatively less well researched, i.e. ordinal and nominal. Therefore in accordance with Cowan's (1991) call for research into number development to be both 'broad and comprehensive' in the next chapter I will examine research into young children's understanding of ways of representing number used not just for counting but for a range of purposes. I will also examine research into children's understanding of the meaning and use of associated written number symbols.

Chapter 2

Young children's understanding of number symbols

Introduction

As discussed in Chapter 1 there has been a considerable amount of research into children's understanding of counting and the logic of cardinal number. Research related to children's conceptual understanding of the ordinal aspects of the number concept is also relatively well developed. In contrast to the amount of detailed research in these areas, less attention has been paid to children's understanding of the application of number to everyday practical situations, i.e. the extent to which they are able to use number for a purpose in the course of communication in everyday contexts. This is to be the focus of a substantial part of the research in this study.

While the main focus of children's early number development has been on cardinal number, it is necessary for research to explore the range of different uses that apply. Number is used in the adult world for a wide range of purposes:

- Nominal: as a means of identification. "Number ten" could apply to a participant in a performance or sport, to listed items, a bus, a residence or to buttons on equipment.

- Ordinal: stipulating the order in which instructions are to be carried out, the order in which customers or patients will be attended to, or the results of a competition.

- Cardinal number: such as when noting the numbers of items, events or people required, allowed or displayed. Cardinal number is also linked to measuring units for time space and age, and to currency units such as tokens, coins and notes.

- Measurements: when requesting, recording, stipulating or limiting imperial or metric measures.
- Value: when requesting, recording, stipulating or limiting the value of money or tokens.
- Time: when referring to or stipulating periods of time, including reference to the clock and the calendar.

Each of the above constitutes a form of communication that can be conveyed in oral and/or written form.

Young children's understanding of the way number is represented in written form was investigated by Allardice in America and by Sinclair & Sinclair in Switzerland during the 1970's, as outlined below, and their work has been influential in the initial development of this line of enquiry. Other pieces of research have taken account of children's oral responses, however one aspect which does not appear to have been fully explored is the relationship between how children communicate in written form with how they might communicate orally. This chapter will therefore also review work on children's understanding of the communicative role of number through oral as well as written and pictorial expression; in each case the cardinal, ordinal or nominal purpose of the number under examination is specified.

2.1 Written representations of cardinal number

Early research on written number was limited to the young child's ability to differentiate between letters and numerals, such as the work of Clay (1975), Lavine (1977) and

Ferreiro & Teberosky (1982). Over the last twenty years of the twentieth century however there has been some growth in interest in this aspect of children's mathematical development.

As the reports in Chapter 1 show, young children learn to recite the natural number sequence and to count the cardinal quantity of sets during the pre-school period, with varying degrees of competence. Counting is a 'tool' to solve the problem of 'how many?' and the cardinal number of the set is a symbol which can be expressed in either oral or written form. This chapter will examine both the written and oral expressions of cardinality as well as other uses of number by children.

On starting formal school at the age of four or five in Britain, or the age of six or seven in other parts of the world, the writing of numerals constitutes a key part of the mathematics curriculum. As discussed above, number symbols or numerals are used by adults in everyday life for a wide range of purposes. In the early years of education however young children are expected to learn to write numerals principally for the purpose of recording quantity or cardinal number. Nevertheless there has until recently been surprisingly little research into the sense that children make of this mathematics task at the point at which it is introduced to children on entry to school.

One of the methods researchers have used to investigate children's understanding of cardinal number has been to present the child with situations in which the quantity of a set of objects needs to be recorded, and to observe whether he spontaneously uses numerals for the purpose of representing cardinality in 'visual' form.

2.1.1 Allardice

Allardice (1977) in the U.S.A. was one of the first to investigate pre-school children's ideas about how to represent number in written form. Her research was designed to uncover whether conventions such as formal symbols and writing from left to right were already used by children prior to starting school, or whether children used their own forms of representation. A total of 81 children from the New York area were studied: 20 children aged between 3:4 and 4:3, and 20 aged between 4:4 and 5:3 were from play groups and day care centres; 21 children aged between 5:4 and 6:3 were from a school kindergarten and 20 children aged 6:4 and 7:3 from the first grade of the same school.

Allardice's study investigated the ability to represent four concepts: cardinal quantity, relative quantity, addition and subtraction, and temporal order. For the purposes of this study only the first of these, the ability to represent cardinal quantity, will be discussed in detail. Each child was seen individually three times; the initial session consisted of screening tasks to establish whether the cardinal concept was understood at a simple concrete level.

During the second session a toy dog (Snoopy) was seated on a table, facing away from two collections of objects: a set of three toy mice and a set of four buttons. Each child was then asked to help Snoopy, as he could not see the objects, and to "put something on paper ...to show how many there are." During a third probing session further questions and tasks were set to clarify uncertainties and to ascertain the full extent of the children's understanding. All of the children participated.

Results showed that 9 of the 20 three year-olds and 15 of the 20 four year-olds made 'adequate' representations; these included pictures, circles or tallies to indicate cardinal quantity. One group of children explained that either they or Snoopy knew the meaning of the marks, e.g. "It says we have three mice."

All of the five year-old kindergarten children and all of the six year-old first graders were classified as having made 'reasonably adequate' representations; they used either numerals or informal symbols such as tallies. This group did not use letters 'improperly'. The use of numerals was seen to increase with age. Interestingly, only half of the four year-olds who had previously demonstrated that they could write numerals actually used them spontaneously in this task. 16 of the five year-olds and 19 of the six year-old first graders used numerals to represent cardinality (Allardice 1977).

2.1.2 Hughes

Allardice's work in New York (1977) was followed up in Edinburgh by Hughes & Jones (Jones 1981), firstly in a large-scale study involving 96 children aged between 3:4 and 7:9. There were twenty-four children in the Nursery group and twenty-four in each of the first three year groups. The children were interviewed individually.

In this first study each child was presented with a collection of items which are familiar to nursery school children: small blocks. The children were firstly shown one, then two, then three, five and six blocks; they were then asked 'Can you *put* something on the paper to show how many blocks are on the table? Particular attention was paid to the adult's use of language. The words 'writing' and 'drawing' were not used in order to avoid suggesting either form of representation to the children, and the word 'put' was used instead.

The marks made by the children to represent the cardinal number were classified according to one of four categories by Jones (1981) as follows:

a) Idiosyncratic: representations without any regularities to denote numerosity; personal pictures or forms of writing that did not convey meaning to the experimenters.

(b) Pictographic: pictorial representations of the shape, size or colour of the blocks, was combined with numerosity (i.e. showing one-to-one correspondence)

(c) Iconic: numerosity combined with personal marks to represent each brick, such as tallies, circles or simple pictures.

(d) Symbolic: conventional symbols used, such as numerals or number words.

The findings (Jones 1981), showed that the pre-school children used the idiosyncratic and iconic modes (29% and 32%) rather than pictographic or symbolic forms (24% and 12% respectively). Children aged over 5:0 were more likely to use pictographic and symbolic responses (47% and 33%). By the third year of school (age 7) the symbolic mode was the most common response.

Hughes (1986) later extended this work to study young children's understanding of the role of symbols in a second, more purposeful task. Hughes devised the 'Tins Game' which required the children to understand the purpose of the task - which was to record numerosity as a means of helping them to remember later how many items there were. This task required the children to record or 'communicate' number to themselves and the adult by the use of written representation.

In the 'Tins Game' fifteen children from a nursery class and ten children from the first year of a predominantly middle-class school were seen individually. Objects familiar to the children were selected: small blocks and small boxes. Four identical small tins containing 0, 1, 2 and 3 blocks were each opened in turn and shown to the children. The lids were then replaced, the tins shuffled, and the children asked to pick out tins containing a given number of blocks. (2... 1... etc.)

After a period in which the children tried to guess, it was suggested that a label on each lid might help. The children were given a pen and asked to "put something on the paper" in order to record and help remember how many blocks were inside. When the labels were completed and placed on the tins they were then re-shuffled and the children asked to identify the tins, to see whether their representations had helped them "to play the game". The children's written responses were then classified in the same way as in the original study.

The findings of this second study corresponded closely with those of the original research. Children used a systematic pattern of representation, with less frequent use of the pictographic mode and greater use of symbols by both four and five year-olds.

2.1.3 Sinclair, Siegrist & Sinclair

The way in which pre-school children learn to represent quantity or cardinality in visual form was investigated further by Sinclair, Siegrist & Sinclair (1983) with 45 children in a Swiss kindergarten. As Swiss school entry is not until the age of 7 the study included children aged between 4:0 and 6:11, with 15 children at each of three ages, 4, 5 and 6.

The children in the study were provided with collections of objects with which they would be familiar: pencils, rubbers, balls and toy cars. They were given a pencil and paper and asked to "note what is on the table". As in Hughes' study, the words 'writing' and 'drawing' were not used to avoid suggesting to the children any particular form of representation' the word 'put' was used instead.

They were then directly asked to represent numerical quantities, e.g. "Could you write down 'two houses'". They were then asked to explain or read back what they had written.

The findings showed that children used more than one form of graphical representation. The youngest children's attempts included marks or 'scribbles' which may be termed 'idiosyncratic'; seven of the fourteen 4 year-olds used this form. Slightly older children focused on either the nature of the objects (pictographic) or the quantity, such as with the use of tallies (iconic); 9 of the 14 four year-olds use this mode, with a mean age of 4:8. In this research representations which included the use of numerals were classified in three categories. Some children used several numerals to represent quantity, possibly in attempts to either record the counting procedure, as in "1, 2, 3, 4", or the number of counting steps used in reaching the cardinal number, as in "4, 4, 4, 4".

The 'correct' use of number symbols and the written names of the number symbols to represent cardinality [e.g. "four"] was again found to develop with age; twenty-six of the forty-five children used numerals appropriately, with a mean age of 6:0. Sinclair, Siegrist and Sinclair (1983) made a similar point to Hughes in noting that even when young children are capable of reading or writing numbers, they often do not use them in situations which require numerals to be written for communicative purposes. The findings were similar to those of earlier tests by Hughes & Jones (1981).

The study of representation conducted by Sinclair et al (1983) with children aged between 4:0 and 6:11 provided interesting information about the knowledge of children in a Swiss pre-school of the same age as those receiving formal education in Britain, including some of whom are subject to statutory SATs assessment.

Sinclair et al (1983) report that 15 of the 17 six-year olds in the Swiss pre-school setting used numerals or written number names appropriately. However, among the children in Hughes' Edinburgh study who were aged six and had received at least one year's school instruction in the infant school, only 12 of 24 used written numerals to represent cardinality.

The six year-olds in the Swiss pre-school study (Sinclair et al 1983) can be expected to have received less instruction of a formal nature in using and writing numbers, and yet nearly all spontaneously used numerals, compared to only half of the children in formal school. Although the small numbers and differing cultural backgrounds of the children involved in the two studies do not make it possible to make a statistically valid comparison, these are interesting findings. This is a cross-cultural observation which raises further questions about universal development and the relative significance of support provided by the home and the school for children's understanding of written number.

2.1.4 Munn

An experiment by Munn (1994) mirrored that of Hughes and Jones (Jones 1981). Children in a Scottish pre-school were introduced to a version of the 'Tins Game'. Mean ages were 3:10 at the start of the study and 4:7 (of school age) at the end of the study. At the start of the year, when they were asked to represent quantity of items, 18 of the 46 children were found to use iconic methods of representation. By the end of the year Munn (1994) found that 23 of the 46 children were using numerals to record the number of blocks.

2.2 Oral communication about cardinal number

Research in mathematics to date has concentrated primarily on young children's understanding of mathematical concepts, skills and applications. However, in test designs which assess understanding of a range of concepts (i.e not always in mathematics) through the children's spoken responses, problems have been encountered when children have a grasp of number but are unable to express it orally, whereas in a practical situation they are able to operate on a task in a way which demonstrates their understanding. As a result much research in developmental psychology has actually avoided reliance on complex oral responses for fear of underestimating the child's understanding of a given concept.

2.2.1 Oral representations of cardinality

Counting is a tool to solve the question of 'How many?', but as Vergnaud (1982) notes, the cardinal number of a set is a symbol which can be expressed in either written or oral form. Learning the count words of one's community is seen by many as integrally related to acquiring language (eg. Barrett 1986; Saxe et al 1987; Durkin 1993) yet the majority of research to date has focused particularly on conceptual underpinnings and written representations - and not on oral representation.

We have seen in Chapter 1 that in the early stages the skill of 'counting' or reciting numbers does not always reflect an understanding of cardinality; Hughes' (1986) for example shows that whereas young children may be able to count a set of objects and orally report a cardinal number, they may not always have an appreciation of quantity or cardinality. Fluck & Henderson (1996) found that children's oral representation becomes increasingly explicit as the child develops the concept of cardinality, i.e. at the point at which the principles of one-to-one correspondence and cardinality can be assessed.

However Bryant (1997) found that when told the amount of sweets given to one recipient, none of the 4 year-olds used this oral information on cardinal number to make the inference that the other recipient must have the same amount. Cowan (1987a, 1987b) also found that children aged 3 to 7 did not use oral cardinal number when asked to make a comparison or relative number judgement between two sets of objects, even when directly encouraged to count; they relied instead on visual cues.

It may be that when children (successfully) undertake non-verbal tasks it is not easy for them to translate their understanding into number words. Bryant (1997) concludes that children do not at first apply their understanding of cardinality to number words. Dowker & Cowan (1998) suggest that the results of their research imply that young children's development of verbal reasoning about number is 'protracted' and that the reasons for this 'remain to be established' (p.20).

2.2.2 Conceptual knowledge and the oral communication of cardinality

Wynn (1990, 1992) gives the average age at which children develop cardinality as 3 years 7 months. Frye, Braisby, Lowe, Maroudas and Nicholls (1989) set this at about one year after learning to count objects aloud which Fluck &

Henderson (1996) estimate to be at around 4:1. According to Karmiloff-Smith (1992), when 3 and 4 year-olds are asked 'how many?' after counting and they then repeat the count, it is because the counting routine is a representation in which implicit knowledge is embedded, even though these components cannot be accessed separately when testing.

In the study by Fluck & Henderson (1996) with children aged 3:6 to 4:6, the tasks 'Count the objects' and 'How many?' both required children to spontaneously repeat the last word of the count. The findings showed that children of this age rarely repeated final count words spontaneously and most had to be prompted at the end of a count. Fluck & Henderson (1996) claim that this demonstrates the child's construction of a principle, consistent with the 'skills before principles' perspective of Briars and Siegler (1984; Siegler 1991a). Furthermore Fluck & Henderson (1996) suggest it is possible that the children who 'grab' and yet are able to count, do understand cardinality but do not use counting due to other reasons, such as lacking 'utilisational competence' (Gelman et al 1986).

When working with such young children it is generally a difficult and sensitive matter examining whether children have a particular skill or understanding. If the child does not convey understanding this could be for several reasons:

a) they have not acquired the understanding or skill

b) they have acquired it yet do not respond to the adult's questions or prompts to demonstrate it, either on the occasion they are tested or in the longer term

c) they have acquired it but do not know when to apply it ('utilisational competence'; Gelman et al 1986).

It may be the case that the children in Hughes' Tins Game (1986) who did not make use of numerals once they had counted did not know that numerals would be relevant to use, or whether they chose or preferred not to do so. The findings of Fluck & Henderson (1996) showed that most 3 and 4 year-olds had to be prompted at the end of a count by the adult before there was last word repetition (LWR), and that once prompted the older age-group (4:2 to 4:6) showed a marked increase. The same may apply to prompting children to write the number, but there is little research evidence of this.

The move to avoid relying on children's use of spoken language in test situations, as discussed above, may have resulted in less attention being paid to spoken language in the field of early mathematics learning, including the extent to which children are able to spontaneously communicate their understanding of number verbally. However for whatever reason, early number research to date appears to have not focused to a great extent on young children's oracy and mathematical language per se.

This is in marked contrast to research in the field of reading development which is embedded in the theory of children's spoken language and its relationship to progress in reading. Hurford (1987) notes that numerals have not attracted much attention from linguists, nor been part of major intellectual debates.

2.2.3 The relationship between oral and written representations of cardinal number

This section will focus on research into children's conception of number as reflected and expressed in speech and in written form.

Much appears to depend upon the child's understanding of the symbolic language used in the teaching situation, and on whether new mathematical language becomes part of the child's own linguistic and conceptual framework. If this is not the case, then superficial mathematical procedures (as in the case of using spoken and written symbols) are likely to be adopted without an understanding of the underlying concepts and meanings.

Studies of children's 'intuitive' understanding of mathematics (e.g. Hersocovics & Bergeron 1894) have focused on the mathematical processes that children use and on how closely this is related to their grasp of the associated mathematical language. For a child to make sense of a mathematical activity, she must understand both the activity and the language. As Donaldson states 'the child does not interpret words in isolation - (she) interprets situations' (1978 p.88). Resnick's (1992) framework for the development of numerical thinking includes a stage in which numbers eventually become the object of thinking and reasoning; Resnick considers this stage to be influenced by the process of instruction and formal representation.

Tests carried out by Cowan show that children aged 3 to 7 do not make use of oral cardinal number when making number judgements between two sets of objects, and even when directly encouraged to count they rely on visual rather than oral cues (Cowan 1987a, 1987b; Cowan et al 1993). As outlined earlier in Chapter 1, Dowker & Cowan (1998) conclude that the development of young children's understanding or reasoning about number is protracted and that the reasons for the delay have yet to be identified. Their findings also suggest that children's success in their non-verbal task is either inaccessible or not 'readily translatable into number words' (p19).

2.3 Written numerals for non-cardinal purposes

Most of the considerable amount of research carried out into children's understanding of number has been centred on their understanding of cardinal number. This is in contrast to the range of uses of numerals in the adult world.

2.3.1 The range of uses and contexts for numerals

Number symbols or numerals are used as a form of communication in the everyday environment for a wide range of purposes and in a variety of contexts. These uses may be related to:

- Nominal uses: numerals which are not related to logical aspects of number but are provided for identification purposes. These include house numbers, numbers on push-buttons (telephone, entry phone, remote-control unit etc.), numbers on buses to denote the route, or on sportswear to identify competitors.

- Ordinal number, such as: the signs on a mini-golf circuit denoting the order in which the game is to be played, the tags allocated to patients as they arrive at a doctor's surgery, and tags allocated to customers queuing at a delicatessen counter.

- Cardinal number, such as on personally written lists, the notice on a bus stipulating the number of standing passengers, or on the contents section of a food packet.

- Measurements on signs and labels representing imperial or metric measures.

- Value: the use of a numeral to represent a value on money or tokens, such as a "5" to represent five pounds on a note or five pence on a coin. However in the former case for example there are not physically five pound-coins present, and the numeral 5 indicates that the note is 'to the value' of five pounds which is more than one pound. This may arguably be regarded as a 'nominal' use, i.e. for the purpose of identification rather than as a cardinal number, although the identifier '5' would come after '1' in a number sequence. Numerals which represent monetary values therefore present children with several difficulties.

2.3.2 Research into children's awareness of the different purposes of written numerals

The above are examples of numerals which young children are likely to encounter in the course of everyday activities in and around the home and pre-school. Few studies have investigated young children's understanding of printed numerals with purposes other than cardinality.

The degree to which young children develop an awareness of cardinal and non-cardinal numerals has been researched only as recently as the the mid 1980's when Sinclair & Sinclair (1984) were among the first to examine in more detail what it is that children know about numerals in their everyday environment.

2.3.2.1 Sinclair & Sinclair

In order to investigate how children 'interpret, understand or read' some of the printed numbers that surround them, Sinclair & Sinclair (1984) conducted a study among 45 Swiss children in Geneva. Subjects were aged between 3 and 6 years; this included 15 four-year-olds, 15 five-year-olds and 15 six-year-olds, all of whom were from a French speaking kindergarten where no formal instruction had been given in either language or number.

In the study each child was shown a set of ten clearly drawn coloured pictures displaying numerals in the following contexts:

on a birthday cake

on a bus and a bus-stop

on house fronts

on grocery price-tags

on a cash register

on a till receipt

on a speed limit sign

on athletes' vests

on a car registration plate

on a lift

The examples were selected as representing the forms in which numerals are most commonly used in the child's environment, i.e. as cardinals, ordinals, to denote price and speed, and nominally as 'number labels'.

The children were asked to describe the meaning or purpose of the numerals. Their responses were classified as follows:

i) no response

ii) simple naming of the numeral

iii) as a name tag

iv) the global or general function of the numeral, but without clear detail

v) the specific purpose served by the numeral, including detail such as quantity, order, classification, or one-to-one correspondence.

Sinclair & Sinclair (1984) found that even where children were unable to correctly name the numeral, they were often able to describe its purpose in context. The ability to name the specific functions of the numerals increased with age with a clear developmental trend towards more adult-like responses. Sinclair & Sinclair report that the majority of children in the group provided responses which combined 'global or general functions with specific functions', but this decreased with age.

Older children were more likely to describe the specific purpose served by the written numeral. Sinclair & Sinclair (1984) suggest that their findings show that the children possess 'a wealth of information and knowledge about the communicative function of numerals' (p70).

However, responses vary significantly with type of item. The ranking of the easiest to most difficult item is as follows:

1) cake

2) lift

3) bus

4) price

5) receipt

6) speed limit

7) athletes' vests

8) bus stop

9) registration plate

The authors point out that the two items recorded as easiest, the cake and the lift, have visual clues provided by the context. For example, five candles were in place on the cake. There were six buttons on the lift panel, from the ground to the fifth floor, arranged in a way that may support children's understanding. These contexts may therefore help the children to construct or extract the meaning, whereas the items found less easy have no such visual clues.

Nevertheless, Sinclair & Sinclair (1984) find that in many instances these pre-school children show the ability to separate the specific purpose of numerals in one particular setting from the very wide range of other possibilities within environmental number.

2.3.2.2 Summary

As the above research into cardinal applications of number shows, children appear to have some difficulty in grasping this particular meaning of numerals. Although the recording of cardinal number features highly in the pre-school and early years school curricula, young children find its purpose to be problematic and slow to grasp. Durkin (1993) also comments on young children's 'very slow' development of an understanding of quantity.

Durkin (1993) reports a 'widespread lack of understanding' and 'gradual comprehension' of the function of printed numerals, yet where research into numerals has taken a broader view, it appears to indicate that young children have a differentiated understanding of their purpose.

As has been discussed, British and Swiss pre-schoolers have a better grasp of numerals where they are used for purposes other than cardinality. These purposes include nominal or labelling uses, yet these applications are aspects which feature far less frequently in the early years curriculum or psychological research.

In view of these contrasting findings and in order to investigate this trend further I will examine young children's understanding of numerals, or number symbols, within the broader framework of symbolism in general.

2.4 Children's use of symbols

Hiebert (1988) discusses two principal types of written symbols taught in primary mathematics:

i) symbols to represent *quantities* of objects (eg.3, 1/2)

ii) symbols to represent actions or *operations* upon quantities

Young children's difficulties in both these areas have been discussed above in relation to their efforts to represent quantities of objects. Difficulties are also reported in the teaching of mathematics when it is noted that older children frequently experience problems in adopting and using both symbols and algorithmic procedures (e.g. Resnick 1987).

This section will examine both the process by which theorists claim children come to learn about symbols, and the possible causes of young children's frequent difficulties in using symbols in mathematics.

Symbols, meaning and concepts

Discussions in the constructivist literature concerning the process by which children come to use symbols, identify complex relationships between an understanding of symbols and the underlying concepts and the meaning associated with the symbol. Both of these aspects will be examined in the following sections.

2.4.1 Symbols and conceptual underpinning

Any sign or symbol is invented by and within the context of a specific society and culture, therefore the meanings and uses of the symbol are passed on by that culture. A further branch of research is beginning to examine factors involved in this process, and such research will contribute to the complex picture of how and what children come to understand when they read and write symbols in mathematics.

Sinclair & Sinclair (1986) draw attention to the 'conceptual content' which underpins our symbol system. Conceptual knowledge, or 'theoretical' knowledge as it is termed by Murray & Sharp (1986), is a form of knowledge 'saturated with a rich network of relationships of many kinds' (Carpenter, 1986).

Vergnaud (1982) defines representation as a symbol system involving signs, and describes concepts and symbols as 'inextricably linked'. Vergnaud (1982) goes on to describe the process by which children acquire conceptual understanding alongside symbol use as involving:

a) the formation of relations or 'relational invariants'

b) an analysis of their properties

c) the building up of further relational invariants

Hiebert (1988) suggests that the following constitute major components of the process by which children learn about mathematical symbols and numerals:

- Connecting individual symbols to referents of quantity. As children cross 'bridges' between the two, specific meanings are created or constructed for number.

- Connecting individual symbols to referents of quantity: as children cross 'bridges' between the two, specific meanings are constructed for number.

- Connecting individual symbols to referents of action or operation on numbers. Here the symbols allow the proposed operation to be conceptualised or visualised 'in the mind's eye'. [It should be noted that no end result of the operation is pre-supposed at this stage.]

- The subsequent cognitive stage of developing 'symbolic procedures'. This depends upon consolidation of the connecting process, when the child is able to observe actions on referents (eg. objects) and to record or 'parallel' this action with symbols (Hiebert, 1988 p.337). Vergnaud (1982) suggests that learners' grasp of concepts is reflected in the way in which they make use of symbols.

2.4.2 The attribution of meaning to symbols

Research findings characterise the nature of young children's learning as embedded in the concrete (e.g. Piaget, 1952) and 'enactive' in its approach (Bruner, 1966). However, Durkin (1993) challenges platonic views of mathematics as "transcendental logic - pure knowledge .. independent of the human agent and waiting .. to be discovered" (p.135).

Clearly there is a relationship and interaction between the conceptual structures children develop for number and children's subsequent encounters with symbols representing a range of meanings and uses of number in varying contexts. The complex picture regarding the development of the concepts, language, skills and applications of number has already been raised. This section aims to examine theories as to how children come to develop meaning (for symbols) per se. The relationship between children's early, existing concepts of number and the social meanings subsequently acquired for associated symbols is a matter for consideration in its own right.

Light & Perret-Clermont (1989) identify growing interest in research into:

i) interpersonal and discursive cues

ii) the 'social mechanisms' of cognitive change

iii) the 'pragmatic, inter-subjective agreements-in-meaning' in young children's cognitive development

Recent studies suggest that 'everyday' situations in which mathematical activities are embedded hold social meaning for young children and provide 'referents' for the language and concepts that they experience and learn. Putnam sees such referents as 'socially fixed' (1989, p.25).

In this way the context facilitates a social 'process' of learning about number in which children discover 'what other people mean' (Durkin, 1993). Weaver (1982) similarly describes children deriving 'interpretation and meaning' from the instructional tasks they are given. Solomon too sees children's learning about number as a 'social practice' of discovering of 'how and when to use numbers' (Solomon, 1989). The use of numbers encompasses both the oral and visual representation of number - including number symbols.

According to Durkin (1993) this social process then becomes internalised and the child re-constructs the social meaning for herself, making it her own. Meaning also serves to provide scaffolding which enables the child to 'make meaningful sense' of the activity (Donaldson 1978).

When the social activity holds meaning for the child it is seen by Donaldson (1978) to provide a form of scaffolding which enables the child to make sense of the activity. According to Donaldson, situations in which practical, meaningful mathematical activities are embedded are, in turn, those which the child is more likely to interpret with meaning, to accept new challenges and to develop new concepts. Donaldson suggests that in such settings the child feels confident to formulate personal, and eventually conventional, mathematical procedures on the basis of real understanding.

Hiebert (1988) stresses that when children reach the stage of moving from concrete situations and referents towards the more abstract manipulation of symbols, there is a need for such meaning to be retained and to be 'accessed when needed'. He emphasises that competence with written symbols depends upon accessing meaning from these earlier, more meaningful situations (Hiebert, 1988 p340).

However with reference to the early stages in the process of learning about numbers and number symbols, Hughes (1986) notes the personalised ways in which pre-school children represent quantity, based on their own 'spontaneous conceptualisations', e.g. using one-to-one correspondence with concrete objects. That is to say, when left to their own devices children identify relations between a quantity and an individual symbol (e.g. a numeral), and create their own personal meaning for the symbol - which may or may not differ from the conventional and socially agreed meanings that they have already encountered or later encounter.

2.4.3 Language, mathematical concepts and social meaning

In Durkin's account (1993), language and concepts associated with the everyday early mathematical activities encountered by the child are embedded in contexts which hold 'social meaning'. According to Durkin meaning and number words are 'closely interrelated' in the early stages so that children do not form concepts of numbers 'then map number words upon them' (Durkin 1993, p148). On considering the successful use of symbols at a later stage and the relationship with language, Durkin (1993) suggests that much appears to depend upon the child's understanding of the symbolic language used by the adult, and on whether the new mathematical language *becomes part of his own linguistic and conceptual framework.*

A further difficulty in the young child's encounters with adults' language is that she cannot always know when a familiar word or expression refers to a familiar or an unfamiliar mathematical process. For example when the child hears the word 'three' it may be intended as a count word, as a cardinal number or as the nominal, identifying number on a house or flat - in which case the latter may or may not also represent the house's ordinal position in the street. Putnam highlights the complexity of the linguistic information to which the child is exposed, emphasising: 'looking inside the brain for the reference of our words is just looking in the wrong place' (Putnam 1989, p.25).

As seen in Chapter 1 and earlier in this chapter, observations of linguistic interactions between parents and children reveal that the meanings about number which adults communicate to young children may assign greater weight to some aspects and less to others. In the case of early number this often shows the adult paying greater attention to 'pointing....and uttering' *count* words, while a child often focuses 'solely on the *object*' in a bid to reduce complexity (Markman 1989). (My italics.) The end result of these two processes, according to Markman, appears to be frequently that less attention is paid to cardinality, ordinality, or relations.

The extent to which new, mathematical language and symbolism become part of the child's own linguistic and conceptual framework, and the richness and range of associated meanings, have both been seen to influence the nature of children's conceptualisations. Research evidence appears to show that if the latter are weak, children are likely to adopt superficial mathematical procedures, such as using spoken and written symbols, without an understanding of the underlying concepts and meanings. Hiebert (1988) stresses that competence with symbolism depends upon children accessing meaning from earlier situations and this includes those meanings originally shared by the adult (Durkin, 1993).

There is some empirical evidence (Wearne & Hiebert, 1989) to suggest that there may actually be a fundamental need for formal symbols to be associated with meaningful referents during children's first encounter with a particular symbol, gained through everyday situations which provide a referent for both language and concepts. As discussed in Chapter 1, the relationship between mathematical task completion and the young child's ability to understand the task or the concepts involved appears to be complex. According to Murray & Sharp (1986) the learning of fundamental principles or concepts associated with the task serve to help a learner make sense of and incorporate the new information. In this way concepts and understanding of the tasks can be seen to not only be closely related, but to enhance one another.

Munn (1994) discusses a process of interaction between the child's personal experiences in using a variety of symbol systems through contact with their culture, and their growing understanding of the communicative function of print. Munn sees this interaction as leading to a grasp of different types of symbols. Furthermore, Munn (1994) identifies an association between the children's grasp of symbolic and communicative aspects of number and their understanding of the communicative purpose of text in general. She suggests that this growth in understanding is linked less to maturation and 'more to do with children's contact with their culture' (Munn 1994, p5).

Socio-cultural theories, which also encompass how children themselves view events, focus attention on the role of the adults with whom the child interacts and on adults' provision of scaffolding which encompasses: their use of language; the goals, meanings and implicit concepts which they share with children; the mathematical functions (among others) which are modelled.

2.4.4 Socio-cultural theory; concepts and goals

In Chapter One the debate concerning the relationship between conceptual and procedural knowledge incorporates Sophian's discussion (1998) of the mathematical goals communicated by adults to children in the course of social interaction. Sophian points out that these goals may differ between school and outside school and thereby direct children's attention to different aspects of number. The particular focus or foci provided by the social setting may in turn be viewed as 'constraints' on the child's experience of, and learning about, number.

For example, we have seen how caregivers themselves vary in the range of opportunities they provide for pre-schoolers to count, say 'how many' and to name the items counted, as well as give the cardinal number. Therefore in any future examination of number development, young children's counting procedures, principles and their use of numbers and symbols may valuably be studied in a way which incorporates consideration of socio-cultural theory and children's everyday lives.

2.5 Children's difficulties in using symbols

It is reported that children frequently adopt and use symbolic and algorithmic procedures in mathematics without an understanding of underlying concepts such as discussed above (e.g. Cockcroft, 1982; Resnick, 1987). Durkin (1993) identifies two further areas giving rise to both early and later difficulties:

i) the nature of the context in which a problem is presented

ii) confusion as to the meanings of the words used by adults (p.146)

Both of these ideas have been discussed in the preceding sections in relation to how children learn about symbols. They will be re-examined here in relation to children's difficulties in understanding symbols written by others and to children making use of symbols themselves.

According to Vergnaud (1982) the basis for subsequently assessing whether or not students have acquired the given concept is to assess their ability to solve 'ordinary' problems while using 'natural language' and to observe the way they use the associated symbols. The findings of research which uses this methodology of 'everyday problems' show that young children often fail to see the significance of number symbols for solving the problem of representing quantity (Hughes,1986, Sophian,1988).

Hughes (1986) found that young children who are able to recall and write numerals may not make use of them when directly asked to record quantity (e.g. blocks in the Tins Game) because they allocate a cardinal or number symbol to each discrete item in the set, usually orally, with one-to-one correspondence. This difficulty appears to relate closely to the difficulty of constructing meaning between referents and symbols. Hughes found a systematic pattern of representation which changed with age, with less frequent use of the pictographic mode and greater use of symbols by age four and five. Hughes considers that this improvement is because in the Tins Game the children's attention was not being drawn to the blocks themselves, but to the different numbers of blocks in the tins (Hughes, 1986). In other words, performance improved due to adult help.

Davis (1991) also comments that mathematics is often seen as independent of a context and Hiebert (1988) identifies this emphasis on formal symbols at an early stage without meaningful referents as a key reason for later difficulties in mathematics (e.g. Davies, 1984, 1986; Mason, 1987b). Wearne & Hiebert (1989) endorse this with some empirical evidence that if such meaningful referents are not in place during children's first encounter with a particular symbol it may be 'more difficult to go back and engage them later'. According to Resnick & Omanson (1987), it is also more difficult at a later stage to 'remediate flaws'.

Hughes (1986) similarly points out that whereas pre-school children's own representations of quantity tend to be based on one-to-one correspondence with concrete objects, schools begin with the teaching of the conventional symbolic system using numerals. Sinclair & Sinclair's comment (1986) that the conceptual content of the symbol system is severely underestimated by schools (p.61) is supported by Hughes (1986) who suggests that there is a 'split' between children's 'spontaneous conceptualisations' and the conventional system of numerals as it is taught in schools.

This view is further supported by Hiebert (1988) who claims that competence with written symbols depends upon children accessing meaning from earlier, meaningful situations, arguing that schools' emphasis on practice and routine results in some doubts about children's 'competence with written symbols' (Hiebert 1988, p340).

2.6 Learning to read number symbols and learning to read text

In parallel to research undertaken on early number, linguists have been examining the ways in which young children learn how to talk, read and write. Since the 1970's research has provided insight into the importance of external experiences in children's development of spoken language. Similarly, the ways in which children learn about print from books and their environment, and how they learn about the personal use of writing, are seen to be closely related to the ways in which children develop spoken language (Holdaway, 1979). Surprisingly however, there appears to be relatively little cross-referencing between this research on literacy and research on the development of number, yet this is a path that may prove fruitful to explore.

For example, when examining young children's understanding of when and how to produce written numbers for their own purposes, it may be relevant to refer to broader research on how children learn to write using the alphabet which is the conventional symbolic system of western culture. For example, current research into reading and literacy has led to the contemporary view that these two skills are closely interrelated with the development of language as a whole. (e.g. Miller, 1977).

2.6.1 The development of language and literacy

Much of the work on language development that emerged during the 1960's focused upon the linguistic rules and structures to be learned by the child, and on how the child organises or 'processes' these incoming experiences in order to learn to speak the language using such rules and structures. (See

Chomsky 1965) Since the 1970's, however, psycholinguistic theory has provided insight into how external experiences of language are combined with the support systems provided by both the linguistic environment and by older peers and caring adults. (See Bruner, 1974; Vygotsky, 1978). During this early period the young child is thus often considered to be an 'apprentice' to language (Miller, 1977).

The ways in which children learn about print in the environment, in books, and about the 'personal production' of print in the form of writing, are seen to be closely related to the ways in which children develop their understanding and use of spoken language (Holdaway, 1979). As far as literacy is concerned it is suggested that children observe adults modelling the everyday use of writing, its purposes and its meanings, and eventually begin to try for themselves how to make simple and then 'special' marks, pictures or symbols which can communicate and carry meaning. At first this writing is for the child but is later produced for others who may receive these meanings via the printed word (Clay, 1975; Ferreiro & Teberosky, 1979/82). Hall (1987) stresses the importance of exposing children to the power of literacy as a form of communication from the early stages, avoiding the traditions of meaningless copying or handwriting drill.

The research also demonstrates (Goodman, 1980) that the effective development of literacy depends on the following:

- the child's oral language
- a rich print environment
- the presence of adults who: a) provide positive examples of when and how to write (letters, words and text; b) encourage and support the child; c) read back the child's early attempts at writing.

These findings, based on observations in the domestic environment, form the basis of recommendations for practice in the school setting. Teachers are urged to provide positive and real-life examples of spontaneously 'writing for meaning' often by acting as a scribe for children in order to record in print 'memorable speech' (Rosen, 1989). The intention is to build upon children's 'developing literacy roots' in order to support the development of young children's linguistic capabilities and ensure the development of literacy in future years (Goodman, 1980).

Research by Goodman (1980) among others shows that pre-school children who become successful 'apprentices' in a literate society (Miller, 1977) make the following discoveries through interaction with their care-givers in and around the home:

1) the significance of written language

2) the oral labels used when referring to written language

3) the purposes served by written language

4) the variety of forms used to construct the meanings communicated by written language

According to such research, when young children have observed adults' everyday modelling of writing, its purposes and its meanings, they begin from a very early age to try making simple marks and then symbols which can communicate meaning. At first this 'writing' is for the child, but is produced later for others who may interpret and receive these meanings via the printed word (Clay, 1975; Ferreiro & Teberosky, 1982). The principle is to identify and build upon these 'developing literacy roots' (Goodman, 1980). The development of literacy is thereby shown to depend on adults modelling when and how to write, and reading back the child's early attempts at writing. Hall (1987) stresses the importance of exposing children to literacy as a 'form of communication' from the early stages and teachers are urged to provide real-life examples of writing in order to record 'memorable speech' in print (Rosen 1989).

In her study of early Number in a Scottish pre-school, Munn (1994) identified a significant tendency for children who were aware of text to read back the representations of number they had made, finding a close relationship between children's writing strategies and their counting strategies. Munn claims that this tendency indicates a relationship between the development of 'text awareness' and awareness of writing - in this case writing about number. Munn (1994) notes that as iconic representation, or 'written counting', disappears it is replaced by symbolic representation; thus children's conceptual development advances and the strategic use of numerals to record cardinality increases. Munn describes this as a more 'literal' strategy (Munn 1994, p.16) and suggests that this leads to a mature concept of text as 'communicating meaning'.

2.7 Summary

Much research has focused on concepts and principles, with some studies on children's use of numerals to represent cardinality. It has been suggested that children may develop counting skills alongside conceptual principles just as children acquire speech alongside linguistic principles. Observing adults modelling reading and writing and trying this out informally is seen as a key learning process, yet although pre-school children are aware of environmental numerals, they rarely use numbers to represent cardinality. For example if 3-4 year-olds are not normally expected to recall quantities of objects they may not recognise its importance in the Tins Game. A socio-cultural approach to early Number research has been proposed, however since the work of Sinclair and Sinclair (1984) there appears to have been no research into children's understanding of numerals other than cardinal, e.g. the time, date, money, giving their age, etc.

The research context is now seen as significant, and needs to reflect how young children see the world. For example, familiar situations in which numerals serve a clear purpose may prove more reliable when investigating understanding of numerals. The role of language in the conceptualisation and 'translation' of mathematics is also recognised, demanding a close match between the experimenter's and participant's language, and yet less attention is being paid to the extent of children's grasp of mathematical language and their ability to spontaneously communicate number verbally.

Vygotsky (1962) and Bruner (1983, 1986) stress the inter-related nature of language and thought and the generative role of language yet, in the field of early number the relationship between the child's graphic representations, spoken language and thoughts has yet to be explored. As Hurford observed in 1987, numerals attract little attention from linguistics and have not been part of any major intellectual debate, and this still seems to be the case. In particular, we still know little about the extent to which graphic representation reflects young children's thoughts, described by Vygotsky as 'inner speech', or what children could express verbally about their representations.

The question may be been raised as to whether numeracy skills develop in a similar way to language and literacy skills. As discussed earlier in this chapter, if this is the case then support for number contrasts with that for literacy. In the case of reading, children hear the 'language of print' as texts are read aloud, are often involved in adult domestic activities, (Goodman 1980; Wells

1981), and are thereby exposed to interaction which help form a metacognitive framework about text (Munn, 1994). It may be that for young children there is no such stable framework providing support for an "overall mental set towards...number" (Munn, 1994).

Whether representation of number presents difficulties for most young children in the UK or whether this is a universal difficulty is unclear as most comparative data relates to arithmetic scores and the education of older children. There is as yet little data on pre-school children's achievements in other cultures which would be needed in order to examine universal development. In summary, although research in developmental psychology in the field of early number is expanding to embrace broader theories, progress is slow and fragmented. The need to add to the evidence base provides the rationale for the design of the number experiments in this study.

Chapter 3

Research design and methodology

3.1 The rationale and focus of the study

The empirical tests which follow aim to examine how young children develop a grasp of number symbols used for a range of purposes. The tests were replicated across three countries for the purposes of comparison and an indication of any universal features or trends. The tests are designed to access the children's understanding of a range of uses of number in a variety of contexts, such as the purposes of reading numerals on display in their environment, writing numerals for a particular everyday purpose, and using number in speech for different purposes.

The studies are designed to provide further information with regard to the following questions:

- Are young children less likely to use numerals to represent cardinal number (e.g. in writing) because it is beyond their understanding of the concept of cardinality, or because they do not feel prompted to record this as a graphical representation?

- Are they less likely to do so either because they do not understand the cardinal meaning carried by numerals or because (according to socio-cultural theory) they are not aware of numerals used in this way in their everyday environment?

- Are young children aware of numerals generally in their environment, i.e. in any contexts, and do they understand any of their other, non-cardinal, uses?

- Are young children aware of the reasons for writing numerals to communicate a range of different meanings?

- Is there a relationship between the numbers children write, what they say, the meanings they have constructed, and their thoughts as 'inner speech'?

- Can the recent findings on children's responses to certain research methods be used to improve the design of studies aimed at examining what pre-school children understand about number?

3.2 The research design of this study

The methodology used in this study is designed to take into account what we know about the difficulties that can be encountered when testing very young children, as well as developmental and linguistic theory. Socio-cultural research suggests that even where children participate in social practices guided by adults, such as the young Brazilian street-sellers, they may do so without understanding.

This study is therefore designed to gain access to some aspects of young children's understanding or conceptual underpinnings of number. It is also designed to explore the relationship between what young children actually do, what they understand, what they are in fact able to do or would do spontaneously - if they could be suitably prompted through an appropriate and effective research design.

In the original English study both groups were composed of a balanced distribution of boys, girls, younger and older younger children, and the order of administration of the 3 Phases was varied. This was designed to control for any order effects and to identify any significant gender differences. As no order effects or gender differences were found it was not considered necessary to include such structures into this research design.

3.2.1. The interviewers and settings

It is known to be important that very young children feel as relaxed as possible during testing, as this enables them to be as forthcoming in their responses as possible and to perform at their best. The researchers spent some time with the children in the classroom setting prior to carrying out the experiments, joining the children in their play or everyday tasks, reading stories or joining in with singing rhymes. This allowed the children to get to know the researchers in a familiar setting so that subsequently being asked to participate in the experiments would not cause apprehension. It also allowed the researchers to become familiar with the children and to ascertain their individual levels of confidence, which assisted interaction with the child during the tests. These points are particularly important when working with 3 and 4 year-olds. It had been arranged for the tests to be carried out in a familiar environment, in their own school and either in their own classroom or in an area adjacent to it and within ear-shot of their peers and normal classroom activity.

3.2.2. Materials and tasks

Research indicates that young children are more likely to demonstrate the extent of their knowledge and problem-solving ability when placed in contexts which hold meaning and interest for them. The tests in this study were therefore designed as tasks which might both appeal to the children, so that they would be keen to participate, and to make sense to them as they were based on situations which are familiar in their everyday lives. For example, two of the tasks designed to test for the representation of number relate to shopping, a task which most children may observe or even participate in, and one task relates to a birthday party. The pictures that children were prompted to talk about are of everyday items such as a telephone and bus etc. The tests are named as follows:

a) Tins Game

b) Note to the Milkman or Shopkeeper

c) Fast Food Order

d) Magic Book

e) Party Invitation

The materials were selected to be familiar to the children so that that they might feel relaxed, but also to avoid any 'novelty factors' detracting their attention from problem-solving. These included items from the nursery classroom, such as blocks, pencils, a toy telephone and items encountered by young children in the course of everyday life such as a doll, milk bottles or cartons etc.

Replication of the Tins Game was carried out as the results of the original study by Hughes (1986) have been widely examined and discussed in relation to young children's knowledge of number in recent years. However this test has not been carried out, as far is known, outside the UK with non-English speaking children of a similar age who are nationals of other countries and other developed cultures.

Whereas the Tins Game investigates how and when children represent cardinal number for their own purposes, a similar test is also used in this study to prompt children to use number for communicating meaning to another person. The task of preparing a note for the milkman was devised as it has the cultural dimension of being familiar to British children, because adults use numerals when writing a note to order extra milk. The task was then adapted for participants in Japan and Sweden who were asked to prepare a note for a shop assistant requesting two cartons of milk. This task was designed to provide an understandable purpose for communicating cardinal number, a reason to use numerals. If relevance is indeed an important factor in children's ability to complete the task, we might expect to see very different results to that seen in the Tins Game. Nevertheless, even if a child considers it is relevant or efficient to use numerals, he may not spontaneously *feel prompted* to use a numeral on the note; however, by stating a recipient or audience for the note, the task is designed to increase the validity of the test.

A further task is provided, placing an order in a fast-food restaurant, which also has the purpose of stating how many items there are, but in this instance the number is communicated in speech.

A 'Magic Book' uses photographs of items which are familiar to the children in each of the three countries as the basis for eliciting their understanding of numerals as they appear in the child's 'everyday' environment.

The final set of tests aims to identify young children's ability to use numerals to communicate both cardinal and non-cardinal meanings to others. These tests are based around the setting of a party as a highly salient event for the majority of children, one which is familiar to all the children involved in the study. The children are shown a blank invitation card and asked to help fill it in, prompting the use of numerals to represent the address, date, time and telephone number.

3.3. Obtaining valid responses

The research designs in this study avoid reliance on complex oral responses because of the associated possibility of underestimating the children's understanding. In view of the immaturity and concentration levels of the participants it was considered restrictive to use interview questions as the sole method of investigating their understanding of numbers. For example, it would be inappropriate to ask young children whether they are 'aware' of numerals in their immediate environment, and would be a leading question. It would also be a complex question unlikely to obtain a response as young children would not wish to participate if they find the situation uninteresting or beyond their comprehension.

Means of response were therefore devised which would assist the communication of such young children. In the Magic Book (environmental numerals) test, for example, participants were shown pictures with numerals omitted where they normally was missing. As young children may not be able to differentiate between letters, numerals and other symbols if asked to name them, linguistic competence was further avoided by allowing them to write or draw what was missing on a drawing.

Questions were open-ended to allow any form of words. It was the task of the researcher to probe and interpret the children's words, or examples given by means of explanation, to access their meanings. The tests also allowed for the researchers' observation of actions, as well as their recording of linguistic responses to interview questions. This approach is based on the Vygotskian tradition that the speech and meanings we encounter in the course of everyday social interactions become internalised, thereby influencing the construction of inner meaning and external actions. The aim was not simply to accept a participant's single immediate response but to allow the child to communicate as much of his or her understanding as is possible on one occasion, in whatever form the child felt appropriate.

3.4. Collation and analysis of results

All responses will be analysed for: a) the ways in which participants represent number used for a range of purposes; b) an indication of the meanings that individual participants hold for cardinal and non-cardinal number; c) any relationship between language, action, meaning and task success. Following collation of the results, patterns and correlations across ages, countries, tasks, strategies and understanding will then be examined.

3.5 The experiments

3.5.1 Method

3.5.1.1. Background

In the initial stages of this research, the experiments were carried out with English children, drawn from contrasting schools. These participants had been matched for age and equal numbers of boys and girls in order to identify any gender differences in the results. The children were allotted to one of three groups, each of which had the tests administered in a different sequence as a means of identifying any order effects.

Overall results showed no gender differences and no order effects. For this reason the sequence in which the studies were administered in Japan and Sweden was not varied and they were carried out in the same order as each other. Overall, an equal number of boys and girls were involved in the experiments in Japan and Sweden.

3.5.1.2 Participants

The English tests were carried out in two well-established nursery schools in Northampton, a large English town in the south-east Midlands. In Sweden the tests were carried out by native speakers of Swedish in three nursery schools in Jönköping, a fair-sized town by Swedish standards, in central Sweden. In Japan the tests were administered by native speakers of Japanese in three nursery schools in towns within the district of Osaka in south-east Japan. Schools were matched within countries to represent working class and middle class areas.

The study carried out in England involved 48 participants aged between 43 and 58 months. In Japan, 71 children participated aged between 42 and 61 months. In Sweden the group numbered 48 and were aged between 36 and 56 months. In each country there was an equal number of boys and girls. The original study carried out in England involved 48 participants who fell into two distinct age groups, each consisting of the same number of participants. It was not possible to obtain the same age profiles when arranging to carry out the tests in Japan and Sweden, therefore the ages of the English cohort were used to define corresponding age-bands; the numbers of participants in each group is noted as these varied. (See Table 4.1 in the next chapter.) Results were collated overall, according to each country and according to age-bands. Allowance was made for such differences in the use of statistical tests.

3.5.1.3 Materials

Tins Game

Four small boxes with lids, six small wooden blocks, a pad of small self-adhesive message sheets, and a fine felt-tip pen.

Note for the Milkman/Shopkeeper (Milk Note)

Two empty milk bottles or cartons, a pad of self-adhesive message sheets, and a fine felt tip pen.

Fast Food Order

A set of materials related to ordering a meal:

- A 40 cm doll (Ronald, the well-known character of an international fast-food outlet) holding a toy telephone hand-set.

- A note-pad, pencil and a second telephone.

- Photographs of fast food items. In each country the foods shown reflected the culture.

In England and Sweden these were:

2 Burgers (Quarter Pounders)

1 boxed Apple Pie

3 Milk Shakes.

In Japan, the foods shown were:

2 rice cakes

1 orange drink

3 fish (not all Japanese children recognise burgers)

Magic Book

The 'Magic Book' contained photographs of three or four everyday items with which the children would be familiar. The four photographs shown to the English and Swedish children were the same: a local bus, a range of domestic telephones, a coin (2p or 1 Krona) and a child's birthday card with an age-badge. The Japanese researchers felt it inappropriate to use the birthday card and bus as they are outside the experience of young Japanese children; instead they were shown photographs of three items: telephones, a 50 Yen coin and a television remote-control handset.

In the first part of the book the items in photographs had a blank space where numerals normally appear. In the second part of the book were the same set of photographs with their numerals 'magically' back in place. The children were also provided with pictures of the same items, printed on paper, and a fine felt tip pen.

Party Invitation

A children's party invitation showing sections for the date, time, address and telephone number, with blank spaces in which the details were to be entered by the child.

3.5.2 Administration

In the UK the tests were administered by the author. In Sweden and Japan they were administered by research assistants. The assistants were native-speaking nationals of Sweden and Japan respectively, selected on the basis of their expertise in working with young children and on their advanced level of English language, particularly in the field of education. This was deemed to be important as the author was able to carry out the research training in person and in direct contact with the assistants, avoiding any minor loss of information that might occur were the training to be conducted through a third person, such as a translator. The training was undertaken using video of the tests conducted in England showing a range of children's responses. This was followed by a pilot in the schools. The same researchers were used throughout the testing period in both Sweden and Japan.

The researchers endeavoured to make themselves known as a friendly and trusted figure by spending a period of time in the nursery classes before commencing the tests. The children were tested individually in a quiet area of the nursery class. The testing of each child was carried out over two days in order to avoid fatigue.

3.5.2.1 Procedure

The Tins Game

The Tins Game was carried out as in the original study (Hughes, 1986). The children were shown each tin in turn; the tins contained 0, 1, 2 and 3 blocks. The lids were then replaced and the tins shuffled and the children asked "Pick out the tin with two blocks in", "Pick out the tin with no blocks in" etc. until all four tins had been discussed. After a short period of guessing, I suggested: "I've got an idea that might help, we could put a label on the top of the tin. Put something on the paper so that you will know how many blocks are inside." Each lid was then removed in turn, in order to view the contents and the child handed the pen to complete the task.

When the children had completed the labels to their satisfaction, the tins were again shuffled and the children asked to identify the number of blocks in each of the tins. The children's attempts to recall or guess were recorded by the experimenter on the recording sheet. The tins were then opened so that the numbers of blocks could actually be seen, and the child referred to his representations. He was invited to: "see whether what you put on the label helped you to play the game".

Coding of graphic representations in the Tins Game

The methods of representation used by each child in the Tins Game were later coded, as in the original test (Hughes 1986) according to the following four categories:

> Idiosyncratic responses: representations without regularities that could relate to the number of blocks present, which could include pictures of other objects, or the child's own form of writing.
>
> Pictographic responses: attempts to represent the blocks pictorially, which may include the shape, position or colour, as well as one-to-one correspondence.
>
> Iconic responses: marks made which represented the blocks displayed with one-to-one correspondence, such as tallies, simple picture or circles.

Symbolic responses: conventional symbols such as numerals or number words.

Note for the Milkman/Shopkeeper (Milk Note)

The two empty milk bottles or cartons were shown to the children, indicating that more milk will be required for when the guests and their parents come to the house. At this point the task was adapted to fit English, Swedish and Japanese cultures. Due to the familiarity of English children with the milkman, they were asked to "put something on the note to tell the milkman how many bottles of milk we want". Swedish and Japanese children were asked to "Put something on the note to tell the shopkeeper how many cartons of milk we want." Each child was then asked: "What does that say, what does that mean?"

Coding of representation in the Milk Note task

The forms of written representation used by the children were then coded according to the same categories used in the Tins Game (see above). Those children who used a numeral several times according to the number of items were initially coded as 'iconic-symbolic', for example, when three blocks were written as: "3 3 3". For the purposes of the study these responses were included as symbolic.

Fast Food Order

The children were told that as another birthday treat they would order items of fast food, and that it would help if they would place the order by phone. The child is asked to phone the assistant who is at the other end of the telephone (behind the screen but visible to the child).

Photographs of the food items were shown to the child. In order to emphasize the correct numbers of items, and to emphasise one-to-one correspondence, the child was asked who would eat each item. He was then asked to: "Tell the lady/man *exactly* what we will need; make sure he/she knows *how many we will need* because the children will be upset if they don't get what they want for tea." Each photograph was then presented individually and in the same order for each child: e.g. two Burgers, one boxed Apple Pie, 3 Milk Shakes. As the child speaks on the telephone the doll was manipulated and appropriate responses made in order to maintain the telephone conversation; no further reference was made by the tester to quantity.

Magic Book - part one:

The children in England and Sweden were shown the photograph of the bus (the TV remote control in Japan) and asked: "Do you think there is anything missing from this bus?" If the reply was negative, the panel which normally bears the route number was pointed out and the children asked: "Is there anything missing from here?" The child was then presented with the drawn picture of the same bus and told that "I think there is something missing" and invited to take the pen and: "Finish the picture. Put in what you think is missing from the front of the bus". If a mark was made, the child was then asked: "What does that mean?"

The children were then asked: "Do you have a telephone in your house?" or: "Do you know anyone who has a telephone in their house?" (Suggesting Grandmother/Auntie/friend etc.) The printed photographs of common types of telephones were presented and the child asked: "What kind of phone do you/they have?" The child was given time to examine the chosen telephone and then asked: "Do you think there is anything missing from this telephone?" If the reply was negative, the area which normally bears the dialling numbers was pointed out and the child asked: "Is there anything missing from here?" The child was then presented with the drawn picture of that particular type of telephone and told that "I think there is something missing" and invited to take the pen and "Finish the picture. Put in what you think is missing from the phone". If marks were added, the children were then asked: "What do they mean?"

Next, the children were shown a printed picture of the tail side of a coin, with the numeral missing. They were then asked "Do you know what this is?" followed by "Do you think there is anything missing?" If the reply was negative, the area which normally bears the numeral was pointed out and the child asked: "Is there anything missing from here?" The child was then presented with a drawn picture of the coin and told "I think there is something missing" and invited to take the pen and "Finish the picture. Put in what is missing from the coin." If a mark was made the child was then asked: "What does that mean?"

Finally, the children were then shown a child's birthday card with a blank badge and asked: "Is there anything missing from this card?" If the reply was negative, the badge was then pointed out and the child asked "Is there anything missing from here?" The child was then provided with a circular sticker to be placed onto the badge and invited to take the pen and "Finish the badge. Put on what is missing." If a mark was made the child was then asked: "What does that mean?"

Magic Book - part two

The children were then asked whether they thought that any 'magic' could have happened, i.e whether anything missing from the pictures could have returned. They were then shown part two of the book containing the same pictures of the bus, the phone, the coin and the birthday card as in the original set, but with the missing numerals in place. The children were then asked of each number label in turn: "What does this mean? Why is it there?"

Party Invitation

The children were shown a pre-printed child's party invitation card. Each heading on the invitation was explained to the children, and their responses discussed.

Each child was then asked: "What do you have to put on the invitation? What do you have to put here . . .?" in each section in turn. Observational notes were also made on the child's oral responses on a recording sheet, in order to match these with the graphic representations they might subsequently make. Each sub-heading on the invitation was pointed to and read aloud to the child once again; he was handed the pen and asked to: "Have a go at filling it in, writing it".

On completing their representations, the children were asked to provide an oral interpretation and meaning for their representations: "What does that say? What does that mean?", and their responses were noted on the observation sheet.

Chapter 4

Results of tests involving use of cardinal number: the Tins Game, Milk Note and Fast Food tasks

The tests carried out in England involved two discrete age groups of children, each consisting of the same number of participants. The younger group contained 24 children aged between 42 and 49 months and the older group consisted of 24 participants aged between 55 and 60 months. As it was not possible to obtain the same age profiles when carrying out the tests in Japan and Sweden for the purposes of comparison, participants' results for all tests were allocated to age bands as shown below in Table 4.1. The numbers of participants in each group is noted as these varied; allowance has been made for such differences in the use of statistical tests. These age bands apply to the results of all of the tests, i.e. those presented in this chapter and those that follow.

Table 4.1 Participants' age groups in the different countries N =167

Age group	Country	N	Age range (months)	Mean age (months)
1	Sweden	8	36 - 41	39
2	England	24	43 - 49	46
	Japan	10	42 - 49	46
	Sweden	17	42 - 49	45
3	Japan	34	50 - 54	52
	Sweden	10	50 - 54	52
4	England	24	55 - 58	57
	Japan	22	55 - 60	57
	Sweden	13	55 - 58	56
5	Japan	5	61 - 62	61

In sections 4.1, 4.2 and 4.3 of this chapter, results for the Tins Game, Milk Note and Fast Food Order tasks will be presented.

4.1 Representation of cardinal number in the Tins Game

The representation on each child's label for each tin was classified as either Idiosyncratic, Pictographic, Iconic, Symbolic or Blank. Table 4.2 shows the overall frequencies of representation according to the number of blocks in the tins among the different nationalities.

Table 4.2 Overall frequencies of different forms of representation used in the Tins Game N = 167

Contents of tin:	Blank	Idio	Pic	Ic	Sym
No blocks	49	65	0	0	53
One block	-	31	22	47	67
Two blocks	-	32	23	50	62
Three blocks	-	34	20	48	65
Overall	49	162	65	145	247

KEY to forms of representation:
Blank = left blank or no response Idio = idiosyncratic
Pic = Pictographic Ic = Iconic Sym = symbolic

As table 4.2 suggests, the frequencies of the types of representation for the empty tin were different from the others, with 49 of the 167 participants choosing to leave the label blank, greater use of idiosyncratic and less use of symbolic than on tins with 1 to 3 blocks. On tins 1 to 3 symbolic was used most frequently by around 65 of the 167 participants, followed by about 50 using iconic; less use was made of idiosyncratic at around 32 and pictographic at around 22.

According to Cochran Q tests the frequencies of the forms of representation used on the tins did not vary significantly according to whether there were one, two or three blocks, (idiosyncratic, $Q = 0.5$, $df = 2$, ns; pictographic, $Q = 1.2$, $df = 2$, ns; iconic, $Q = 0.6$, $df = 2$, ns; symbolic $Q = 1.5$, $df = 2$, ns).

Table 4.3 below shows the overall results for the use of representation across all four tins, with the findings for 0, 1, 2 and 3 blocks combined according to the different age-groups.

Changes in form of representation with age

Table 4.3 Overall relationship between form of representation for 1, 2 and 3 blocks and age in the Tins Game N = 167

Tins with 0, 1, 2 and 3 blocks

Age groups (age in months)	N	Blank	Idio	Pic	Ic	Sym
1: 36-41	8	1	7	0	21	3
2: 42-49	51	14	80	23	50	37
3: 50-54	44	16	27	19	37	77
4: 55-60	59	17	48	23	37	111
5: 61-62	5	1	0	0	0	19
All groups	167	49	162	65	145	247

Among the 8 youngest children it is surprising to see that iconic was the most frequently used form of representation, while pictographic was not used at all and a fifth of representations were idiosyncratic. Conversely, in age group 2, idiosyncratic was used the most often, by over a third of children, followed by iconic and symbolic, with pictographic used the least. Among age groups 3, 4 and 5 symbolic was used the most often while idiosyncratic fell steadily overall to around a fifth in age-groups 3 and 4 and none in group 5.

As this suggests, the type of representation varied with age. Non-parametric tests of correlation confirmed that older children used fewer idiosyncratic representations ($r_s = -{\cdot}33$ $p < {\cdot}01$) and iconic representations ($r_s = -{\cdot}\ 26$, $p < .01$) but more symbolic representations ($r_s = {\cdot}42$, $p < {\cdot}01$).

The trend for pictographic was not significant (r_s = - ·02 ns).

Accuracy in reading back the labels

The overall ability of participants to read the labels and recall the number of blocks in each tin was examined in relation to the types of representation they had used. Unfortunately the Japanese children's accuracy in reading back their labels was not recorded. Table 4.4 shows the overall relationship between forms of representation and accuracy for the English and Swedish children.

Table 4.4 Overall relationship between form of representation and accuracy in reading back labels (N =96)

Number of blocks	Blank	Idio	Pic	Ic	Sym	Overall
0	21/22	24/54	-	-	18/20	63/96
1	-	3/23	10/10	35/36	25/27	73/96
2	-	9/23	12/12	34/39	20/22	75/96
3		4/25	7/8	35/37	23/26	73/96
Overall	21/22	40/125	29/30	104/112	86/95	284/384

As can be seen from table 4.4, accuracy in reading back the labels did not vary with the number of blocks in the tins. It did, however, vary with the form of representation used. Whereas accuracy was only 33% for idiosyncratic labels, accuracy levels for all other forms of representation were above 90%.

Overall age trends in accuracy at reading back labels

Results were examined for age-related trends in the accuracy with which labels were used to recall numbers of blocks, as shown in Table 4.5.

Table 4.5 Overall relationship between accuracy in reading back labels for 0 – 3 blocks and age (N=167)

Age groups (age in months)	N	Blank	Idio	Pic	Icon	Sym	Comb
1: 36-41	8	1/1	3/7	-	20/21	3/3	27/32
2: 42-49	41	9/10	25/74	15/16	42/45	16/21	107/164
3: 50-54	10	2/2	4/7	3/3	17/21	4/7	27/40
4: 55-60	37	9/9	12/37	11/11	25/25	63/66	120/148

Key: Comb. Indicates the use of a combination of forms of representation across the 4 tins.

Results within ages for tins 0, 1, 2 and 3 have been combined. All age-groups appear to recall accurately where the label was left blank, whereas accuracy rates when using the four forms of representation show differing patterns of results according to age.

Overall, according to Spearman rank correlation tests, the older children were found to be significantly more successful (r_s= ·202 p< ·05) in reading back their labels. This seems to be due to changes in the type of representations they had used rather than improved performance with the same type of representation.

Separate descriptions for each country

Results for each country in the use of representation were examined separately in order to identify any differences in performance.

England

Frequencies of the types of representation used by English children on tins 0, 1, 2 and 3, are shown below in Table 4.6.

Table 4.6 Frequencies of different forms of representation used by English participants N=48

	Blank	Idio	Pic	Ic	Sym
0 blocks	14	26	-	-	8
1 block	-	14	7	10	17
2 blocks	-	15	10	10	13
3 blocks	-	20	5	8	15
Overall	14	75	22	28	53

Results for the tins with 1, 2 and 3 blocks show a similar pattern. Between 14 and 20 of the 48 children used idiosyncratic forms of representation, between 5 and 10 children used pictographic forms; 8 to 10 children used iconic forms of representation and 13 to 17 used the symbolic form. Pictographic and iconic types of representation were used by between 5 and 10 children, which was less frequent than idiosyncratic and symbolic forms which were used almost twice as frequently, by between 13 and 20 children.

The greatest contrast however is between the types of representation used on the tin with no blocks. Here, idiosyncratic was used more often on the empty tin than on tins 1-3, with 26 children using idiosyncratic compared to between 14 and 20 on tins 1-3. Only eight children used a symbolic representation to label the empty tin, whereas 15 did so for the other tins.

To examine how the type of representation varied with number of blocks, separate Cochran Q tests were conducted for tins 0-3. Only idiosyncratic and symbolic forms of representation were compared; pictographic and iconic forms of representation were not compared as it is a moot point as to whether leaving the label blank for the empty tin should be considered as either iconic or pictographic.

The tests showed reliable variation in the use of idiosyncratic labels, and to some extent, symbolic labels (idiosyncratic, $Q = 15.34$, $df=3$, $p< .01$; symbolic, $Q = 7.78$, $df = 3$, $p =.051$). Subsequent comparisons for just the tins with one to three blocks showed that only the use of pictographic varied according to the number of blocks. However variation in the use of idiosyncratic approached significance (symbolic, $Q = 3.00$, $df = 2$, ns; idiosyncratic, $Q = 5.64$, $df = 2$, $p = .06$; iconic, $Q = 1.00$, $df = 2$, ns; pictographic, $Q = 7.60$, $df = 2$, $p < .05$).

Age-related trends and forms of representation

Age trends were also examined according to the types of representation used on tins 0-3 by the English participants as shown in Table 4.7.

Table 4.7 Relationship between forms of representation and age among English participants

Tins with 0.1 2 and 3 blocks

Age groups (age in months)	N	Blank	Idio	Pic	Ic	Sym
2: 42-49	24	7	48	14	16	11
4: 55-60	24	7	27	8	12	42

As the English group had been selected on the basis of an equal distribution of older and younger children, results for the two age groups shown in table 4.7 allow age trends to be easily examined by eye. On tins 0-3 idiosyncratic and pictographic uses fell by almost a half, from 48 responses to 27 and from 14 to 8 respectively. There was also a small fall from 16 to 12 in the use of iconic. These decreases are accounted for by a major increase in the use of the symbolic form with age, with 11 symbolic responses made by younger children rising to 42 responses made by the older group, almost a four-fold increase.

Spearman rank correlation tests suggest a significant decline in use of idiosyncratic with increasing age among English children ($r_s = -.426$, $p<.001$). There was no overall age related trend in the use of pictographic or iconic forms of representation. As might be expected, the increase in use of symbolic with age was significant ($r_s = .49$ $p<.05$).

Accuracy

English children's accuracy in using the labels to recall the numbers of blocks was also investigated. The results in table 4.8 show how many children were successful according to the type of representation used.

Table 4.8 Accuracy of reading back the labels among English children according to form of representation used

Nr of blocks	Blank	Idio	Pic	Ic	Sym
0	13/14	9/26	-	-	7/8
1-3	-	10/49	22/22	27/28	42/45

As the table shows, 13 of the 14 children who left the label blank on the empty tin successfully recalled that there were no blocks, a high rate of success.

Among the 48 participants, of the 26 who used idiosyncratic on the empty tin, a third were able to recall that the tin was empty. On tins 1, 2 and 3 however, only a fifth of those using idiosyncratic were able to recall tins the numbers of blocks successfully. Almost all of the English children who used pictographic and iconic were successful, as were nearly all children who used the symbolic form on both the empty tin and tins 1 to 3.

Spearman rank correlation tests showed that there was a significant age-related trend among English participants, with older children being significantly more likely to read back their labels accurately ($r_s = \cdot 478$ $p<\cdot 01$).

Japan

Table 4.9 shows the numbers of Japanese children using each form of representation the tins containing 0, 1, 2 and 3 blocks.

Table 4.9 Frequency in the forms of representation used by Japanese children N = 71

	Blank	Idio	Pic	Ic	Sym
0 blocks	27	11	-	-	33
1 block	-	8	12	11	40
2 blocks	-	9	11	11	40
3 blocks	-	9	12	11	39
Overall	27	37	35	33	152

As table 4.9 shows, the pattern of use of representation differs between the empty tin and tins 1-3. This is due almost entirely to leaving the label blank on the empty tin. Apart from leaving the label blank for the empty tin, Cochran Q tests to determine whether form of representation varied with contents of the tin indicated no reliable variation (Symbolic, 0-3, $Q = 3.78$, $df = 3$, ns; Symbolic 1-3, $Q = 0.14$. $df = 2$, ns; Idiosyncratic 0-3, $Q = 1.33$, $df = 3$, ns; Idiosyncratic 1-3, $Q = 0.28$, $df = 2$, ns; Iconic, $Q = 0.00$, $df = 2$, ns; Pictographic, $Q = 0.40$, $df = 2$, ns).

Age related trends in representation among Japanese children

Use of representation on all four tins combined, according to age, can be seen in table 4.10.

Table 4.10 Relationship between representation and age on Tins with 0-3 blocks among Japanese participants

Age groups (age in months)	N	Blank	Idio	Pic	Ic	Sym
2: 42-49	10	4	6	7	5	18
3: 50-54	34	14	20	16	16	69
4: 55-60	22	8	11	12	12	44
5: 61-62	5	1	0	0	0	19

As seen above, the results show only a narrow range of differences between age-groups in the use of blank, pictographic and iconic, whereas over half of the participants in each age-group used symbolic. In the oldest age-group however almost all children used symbolic.

Spearman rank correlation tests showed an association between less use of idiosyncratic with increasing age (r_s = -.225, $p < .05$). There were no significant trends with age in the use of pictographic and iconic forms of representation, but the use of symbolic on the tins showed a significant increase with age (r_s = .27 $p < .05$).

Accuracy

Data and statistics for accuracy in reading back labels are not shown for Japan as unfortunately these tests were not carried out with Japanese participants.

Sweden

As table 4.11 suggests, the type of representation used for zero was different to that used on tins 1, 2 and 3.

Table 4.11 Overall frequencies of different forms of representation among Swedish participants N = 48

	Blank	Idio	Pic	Ic	Sym
0 blocks	8	28	-	-	12
1 block	-	9	3	26	10
2 blocks	-	8	2	29	9
3 blocks	-	5	3	29	11
Overall	8	50	8	84	42

On the tin with no blocks the label was left blank by only a sixth of the 48 Swedish children, whereas over half used idiosyncratic. Results of a Cochran Q test indicated that the use of idiosyncratic on the empty tin was not significantly greater than on tins with 1, 2 and 3 blocks combined, (q = 48.15 p = 0.00).

Overall on tins 1, 2 and 3 they appear to make greatest use of idiosyncratic with a total of 50 uses, and iconic, with a total of 84 applications. The choice of representation is similar on tins with one to three blocks in them, showing no significant trend in the use of idiosyncratic (p =.197 ns), pictographic (p=.6 ns), iconic (0.28) or symbolic (p=.37 ns).

The Swedish children treated the empty tin differently from the tins with 1 to 3 blocks in, and some left the label blank, the most common form of representation was idiosyncratic. Indeed greater use of an idiosyncratic label on the empty tin was the only significant variation in type of type of representation with contents of tin (Symbolic, 0-3 blocks, Q = 1.88, df = 3, ns; Symbolic, 1-3 blocks, Q = 2.00, df = 2, ns; Idiosyncratic, 0-3 blocks, Q = 48.15, df = 3, p < .001; Idiosyncratic, 1-3 blocks, Q 3.25, df = 2, ns; Iconic, Q = 2.57, df = 2, ns; Pictographic, Q = 1.00, df = 2, ns).

Age related trends and the use of representation

Changes in the use of representation were also examined for any changes with age, as shown in table 4.12.

Table 4.12 Age trends in forms of representation on Tins with 0, 1, 2 and 3 blocks among Swedish participants N = 48

Age groups (age in months)	N	Blank	Idio	Pic	Ic	Sym
1: 36-41	8	1	7	0	21	3
2: 42-49	17	3	26	2	29	7
3: 50-54	10	2	7	3	21	7
4: 55-60	13	2	10	3	13	24

The results above suggest that the use of blank was just about constant through the age-groups, and as we know its use was confined to the tin with no blocks in. There appears to be a small decrease with age in the use of idiosyncratic. On the tins with 1 to 3 blocks the use of pictographic is very low in all age-groups and shows little change with age. Interestingly the use of iconic appears greatest among the younger participants as its use falls with age, whereas the use of symbols shows a marked increase among the oldest Swedish children.

Spearman correlation tests confirmed no significant trend in the use of idiosyncratic with age, although Swedish children made significantly more use of it on the tin with no blocks ($Q = 48.15$ $df=3$, $p <.01$). On tins 1-3 there was no trend in the use pictographic; they used iconic significantly less with age ($r_s = -.34$, $p <.05$) whereas in the use of symbolic, participants showed a significant increase with age (Sweden $r_s =.37$ $p<.05$).

Accuracy in reading back the labels

Rates of accuracy for reading back the labels among Swedish children are shown in table 4.13 according to the form of representation used on the label.

Table 4.13 Relationship between form of representation and accuracy in reading back the labels

Nr of blocks	Blank	Idio	Pic	Ic	Sym
0	8/8	15/28	-	-	11/12
1	-	2/9	3/3	25/26	9/10
2	-	3/8	2/2	25/29	8/9
3	-	1/5	2/3	27/29	9/11
Overall	8/8	21/50	7/8	77/84	37/42

As the table shows, on the empty tin all of the 8 Swedish children who left the label blank were successful in recalling the number of blocks, as were almost all those using a symbolic form of representation. On the tins containing 1,2 and 3 blocks accuracy of recall appears to be low among those using idiosyncratic, whereas accuracy is consistently high across all other forms of representation.

However Spearman rank correlation tests showed that overall there was no significant trend related to age among Swedish children in their use of representation to accurately read back their labels ($r_s = -.051$, ns).

Comparison across countries

A comparison between forms of representation used on tins 0-3 was carried out using the results for the two matched age-groups 2 and 4. Table 4.14 shows the frequencies in the use of the different forms of representation according to age group across the three countries.

Table 4.14 Different forms of representation used on the tins with 0, 1, 2 and 3 blocks combined by children in the 3 countries according to age-group

Age groups (age in months)	N	Blank	Idio	Pic	Ic	Sym
2: 42-49	51					
England	24	7	48	14	16	11
Japan	10	4	6	7	5	18
Sweden	17	3	26	2	29	8
4: 55-60	59					
England	24	7	27	8	12	42
Japan	22	8	11	12	12	45
Sweden	13	2	10	3	13	24

The table appears to suggest that leaving the label blank remained similar across the age-groups in all three countries. In England and Sweden there is a marked fall in the use of idiosyncratic and iconic in the older age-groups. In all three countries there appears to be a marked increase in the use of symbolic among the older children.

As the proportions of children in each of the two common age groups varied across country, separate comparisons of countries were made for each age group.

Amongst the younger children, frequency of use of idiosyncratic representations varied with country (Kruskal Wallis chi-square = 6.83, df = 2, $p < .05$). English children used it most often and Japanese least often but only the difference between Japanese and the other children were reliable according to Mann-Whitney tests (England v Japan, U = 56, $p < .025$; England v Sweden, U = 172.5, ns; Sweden v Japan, U = 48, $p <.05$).

Symbolic representation use varied with country (Kruskal Wallis chi-square = 11.69, df = 2, $p < .005$). Japanese children used it more often than English and Sweden children who did not reliably differ (England v Japan, U = 47, $p < .005$; England v Sweden, U = 181.5, ns; Sweden v Japan, U = 35, $p <.025$).

Iconic representation use varied with country (Kruskal Wallis chi-square = 7.56, df = 2, $p < .025$). Swedish children used this form of representation more than either Japanese or English children (England v Japan, U = 117, ns; England v Sweden, U = 120.5, $p < .025$; Sweden v Japan, U = 47, $p <.05$).

Pictographic representation use did not vary with country (Kruskal Wallis chi-square = 2.08, df = 2, ns). So no further tests were conducted.

Amongst the older children, type of representation did not vary reliably between countries (Idiosyncratic, KW = 4.17, df = 2, ns; Symbolic, KW = 0.63, df =2, ns; Pictographic, KW = 0.74, df = 2, ns; Iconic, KW = 1.61, df =2, ns). As there were no overall differences, no further tests were conducted.

Accuracy in reading back the labels

The children's accuracy in reading back the labels that they had written for the tins was examined in relation to the form of representation used, as shown in table 4.15, in England and Sweden only.

Table 4.15 Relationship between form of representation and accuracy in reading back the labels on Tins with 0,1, 2 and 3 blocks

Country	N	Blank	Idio	Pic	Ic	Sym
England	48	13/14	19/75	22/22	27/28	49/53
Sweden	48	8/8	21/50	7/8	77/84	37/42

Accuracy of reading back appears lowest for the idiosyncratic form in both countries, in England about a quarter and in Sweden less than a half of the children recalled this form of representation accurately.

Recall using the pictographic form of representation was very high in both countries, with 7 of the 8 Swedish children and all 22 of the English children recalling the number accurately. Iconic too was high, with 27 out of 28 children in England recalling successfully and 77 of the 84 Swedish children doing so - figures of between 96 and 92 per cent respectively.

The rate of accuracy for symbolic representation was also high in both countries but highest in England with 49 of the 53 children recalling successfully, or 92 per cent; in Sweden 37 out of 42 were accurate or 88 per cent of children. According to a Kruskal-Wallis test, there was no significant difference in accuracy rates between England and Sweden on comparing age-groups 2 and 4 ($KW_{(1)}$) = 1.85, p =.17).

Unfortunately, accuracy tests were not carried out with the Japanese participants. However their accuracy might be expected to be similar or better than those of English or Swedish participants, particularly in relation to symbolic representation which was at between 88 and 92 per cent for England and Sweden. Although the used of symbolic representation did not differ significantly among the older groups in all three countries, in Japan it was used significantly more among the younger age-group than in England or Sweden (England v Japan, $U = 47$, $p < .005$; England v Sweden, $U = 181.5$, ns; Sweden v Japan, $U = 35$, $p < .025$), so the rate of accurate recall among the younger Japanese children would be interesting to explore.

4.1.1 Overview of results of the Tins Game

The tins game was incorporated into this study as it provided a meaningful, although unfamiliar context for children to both make and use graphical representations that distinguish between cardinal numbers. In this task, although the use of numerals was important for successful performance it was not entirely necessary. The task therefore serves the purpose of eliciting children's choice of representation for recording cardinal number for *their own* purposes, i.e. for later recalling the numbers of blocks. The tins game task also allows a comparison of the age at which children use symbolic and other forms of representation in the three countries.

On the tins with 1, 2 and 3 blocks there was an overall decline in the use of idiosyncratic and iconic forms of representation with age. When combined with an increase in the use of symbolic forms of representation, this suggests a general shift towards a more mature form of representation with age. Children in all three countries showed a similar pattern, although among the younger group aged 42-49 months, children in Japan showed greater sophistication by using less idiosyncratic and more use of a symbolic from of representation compared to younger children in England and Sweden. However, children in England and Sweden caught up with the Japanese children in the older group (aged 55-60 months) where there were no significant differences in choice of representation between participants in the three countries.

The results show that on the tin with no blocks the overall picture was different to that seen on the tins with 1 to 3 blocks, with some differences between countries. A third of Japanese participants and a quarter of English left the label blank to represent zero, yet hardly any of the Swedish children did so. It is a moot point whether leaving a label blank could be interpreted as similar to pictographic and iconic forms of representation. As there was no significant increase with age in the use of a blank label to represent zero, it may not be viewed as a more mature way of representing zero.

The study compared participants' responses when reading back their labels to recall the numbers of blocks in each tin with the different forms of representation in order to evaluate their effectiveness. Although symbolic and iconic forms of representation may be considered the most sophisticated forms, pictographic was equally effective in enabling children to recall the numbers of blocks, with all three forms at around 90% accurate. Leaving the label blank was equally effective overall for recall purposes at around 90% accurate. Accuracy rates overall did increase with age, but this appears mainly due to older children moving to a more mature form of representation; in particular, this involved a marked decline in the use of idiosyncratic types of representation.

The low results for symbolic forms of representation including numerals are unlikely to be due to unfamiliarity with writing numbers, as after all the tasks were completed, the English participants were each asked to write their numbers and they were all able to do so. The results on the tins game therefore suggest that although children overall may develop their grasp of when and how to use numerals for representing the cardinal aspect of number at similar rates, Japanese children may do so somewhat earlier than children in England and Sweden.

4.2 Representation of cardinal number in the Milk Note task

The task of writing a note to another person such as the milkman or a shopkeeper was created to be more meaningful or purposeful to young children than the Tins Game. As such it was expected to elicit from participants a more symbolic or conventional form of representation than was used on the tins as this would be more likely to be understood by the person receiving the note. However, the results overall on the Milk Note are somewhat surprising.

Table: 4.16 Comparison of overall frequencies of different forms of representation on the Tin with 2 blocks and the Milk Note N=167

	Blank	Idio	Pic	Ic	Sym
Tin	-	32	23	50	62
Milk	7	59	25	36	40

As table 4.16 shows there is an increase in the use of idiosyncratic when writing a note for the milkman or shopkeeper whereas the use of pictographic remains about the same as in the Tins Game. Use of iconic and symbolic forms of representation, which may be considered as more sophisticated, fell by about a third on the Milk Note. Whether this pattern was confined to younger children was then investigated.

Age trends and representation on the Milk Note

The overall relationship between age and different forms of representation used on the note to the milkman/shopkeeper is shown in table 4.17, where results are shown for each of the five age groups.

Table 4.17 Overall relationship between forms of representation used on the Milk Note and age N = 167

Age group (age in months)	N	Blank	Idio	Pic	Ic	Sym
1: 36-41 months	8	0	2	2	4	0
2: 42-49	51	2	20	6	16	7
3: 50-54	44	3	16	7	7	11
4: 55-60	59	2	21	8	9	19
5: 61-62	5	0	0	2	0	3

The results in Table 4.17 show that hardly any children left the note blank on the Milk Note. The use of idiosyncratic remains about constant overall whereas the use of iconic appears to fall with age. Symbolic representation increased with age, with only a tenth of children in the two younger groups using it and over a third of children in the two older groups using symbols.

According to Mann-Whitney tests the only significant age differences on the note for the milkman/shopkeeper were that the children who used the iconic form of representation were younger ($p < .001$) and those who used the symbolic form were older ($p < .001$). This pattern of age differences is also shown on the Tins task except the children who used the idiosyncratic form of representation on the Tins task tended to be younger.

Overall relationship between representation on the tin and representation on the Milk Note

The overall relationship between children's use of representation on the tin with 2 blocks and on the note for 2 units of milk can be seen below in table 4.18, which shows whether individual children's use of representation differed across the two tasks.

Table 4.18 Overall relationship between forms of representation used on the tin and Milk Note N=167

Representation on Tin 2:	Representation on Milk:					
	Blank	Idio	Pic	Ic	Sym	Totals
Blank	-	-	-	-	-	-
Idio	2	18	5	3	4	32
Pic	1	9	8	4	1	23
Ic	2	19	5	22	2	50
Sym	2	13	7	7	33	62
Totals	7	59	25	36	40	167

Considering the patterns of representation on the two tasks indicates mixed results. Of the children who used an idiosyncratic label on the tin, almost half (12/32) used a more sophisticated form on the Milk Note that might be more readily understood by another person.

However, children who used the more intelligible forms of representation for the tin label were quite likely to use a less intelligible label (blank or idiosyncratic) on the Milk Note.

This prompts questions as to whether this suggests that children move to use a more understandable form of representation when asked to write a note for the milkman/shopkeeper. It seems that the answer is no, as the numbers who did so are outweighed by the numbers who did not.

According to the McNemar Test for Significance of Changes, children were overall more likely to leave the note for the milkman/shopkeeper blank ($p=.016$), and less likely to use a symbolic representation ($X_2=13.08$, $p=.000$) or iconic representation ($X_2=4.02$, $p=.045$). They were much more likely to use an idiosyncratic representation on the note for the milkman/shopkeeper ($X_2=12.29$, $p=.000$). There was no significant difference in the use of pictographic between the two tasks.

Forms of representation used on the Milk Note task according to country

As this task was designed to be more meaningful to young children, it is interesting to explore whether this was the case in any of the three countries.

England

As table 4.19 shows, English children's use of representation was not prompted to become mature with the Milk Note task, with increased use of idiosyncratic and a slight decrease in symbolic.

Table: 4.19 Comparison of forms of representation used on the Tin with 2 blocks and the Milk Note by English children N=48

	Blank	Idio	Pic	Ic	Sym
Tin	-	15	10	10	13
Milk	0	23	2	13	10

Comparing the forms of representation used on the note for the milkman and the tin with 2 blocks using the McNemar test for Significance of Changes showed some differences. English children were a little more likely to use idiosyncratic on the Milk Note, at a level approaching significance (p=.08).

They were significantly less likely to use pictographic on the Milk Note (p=.04) but there was no significant difference in the use of iconic or symbolic. These results are perhaps surprising given the tradition of milk delivery and writing notes for the milkman in England, a tradition which did apply to the children participating in the English study.

Age trends in representation on the Milk Note among English children

Table 4.20 shows results on the Milk Note for English children across the two age-groups of 24.

Table 4.20: Frequency of different forms of Representation on the note to the milkman according to age groups 2 and 4

Age groups	N	Idio	Pic	Ic	Sym
2: 42-49 months	24	12	1	8	3
4: 55-60 months	24	11	1	5	7

When examined for change across age the results in the table show a small increase in the use of symbolic with age among English participants combined with a slight fall in the use of iconic.

However, no reliable associations were found between age and form of representation on the note for the milkman (Iconic, rs = -.14, ns; Symbolic, rs = .21, ns; Idiosyncratic, rs = -.04, ns; Pictographic, rs = -.03, ns).

Japan

Results comparing the use of representation by Japanese participants on both the Tins Game and Milk Note are shown in table 4.21.

Table: 4.21 Comparison of overall use of representation on the Tin with 2 blocks and the note to the shopkeeper N=71

	Blank	Idio	Pic	Ic	Sym
Tin	-	9	11	11	40
Milk	7	22	16	4	22

These results suggest a marked difference in the pattern of representation on the two tasks. The use of idiosyncratic appears to double on the Milk Note task compared to the Tins

Game. The more sophisticated forms of iconic and symbolic representation fall by almost a half on the Milk Note as these forms are used by only 4 and 22 out of 71 respectively on the Milk Note, compared to 11 and 40 respectively in the Tins Game.

The McNemar test for Significance of Changes showed several differences. When compared to the Tins Game, children are shown to be significantly more likely to use blank ($p = .016$) and idiosyncratic ($p = .007$) on the Milk Note. There was no significant difference in their use of pictographic but they were slightly less likely to use iconic, at a level approaching significance ($p = .065$), and significantly less likely to use symbolic ($p = .00$). Whether these findings relate to differences across the age groups was examined further.

Age trends in representation on the Milk Note among Japanese children

Results on the Milk Note for Japanese participants are shown in table 4.22 below.

Table 4.22 Relationship between age and form of representation on the note to the shopkeeper among Japanese children N=71

Age groups (age in months)	N	Blank	Idio	Pic	Ic	Sym
2: 42-49	10	2	4	2	0	2
3: 50-54	34	3	12	6	3	10
4: 55-60	22	2	6	6	1	7
5: 61-62	5	0	0	2	0	3

Table 4.22 shows that the use of blank fell with age. Idiosyncratic was used most by the youngest children with none of the oldest children using this form, whereas pictographic was used consistently by about a third of all participants. Iconic was used by a small number of children throughout but the use of symbolic appeared to rise

steadily, with a third of children aged 4 to 5 years and 3 of the 5 oldest children using symbols.

However, no reliable associations were found between age and form of representation on the note for the milkman (Iconic, rs = -.14, ns; Symbolic, rs = .16, ns; Idiosyncratic, rs = -.13, ns; Pictographic, rs = .13, ns)

Sweden

As can be seen in table 4.23, Swedish children appear to increase their use of idiosyncratic and pictographic when writing the note to the shopkeeper.

Table: 4.23 Overall frequency of different forms of representation used on the Milk Note by Swedish children N=48

	Blank	Idio	Pic	Ic	Sym
Tin	-	8	2	29	9
Milk	0	14	7	19	8

There was less use of iconic on the Milk Note compared to the tin, whereas the use of symbolic did not vary on the Milk Note.

Comparing the forms of representation used on the note for the milkman and the tin with 2 blocks using the McNemar test for Significance of Changes showed some differences. Swedish children showed no difference between the tin with two blocks and the Milk Note in the use of idiosyncratic (.238 ns), with slightly more use of pictographic on the Milk Note at a level approaching significance (p=.063). In the more mature forms of representation, Swedish children used iconic significantly less on the Milk Note (p=.041) and used symbolic slightly less on the Milk Note although this was not significant (1.00 ns).

Age trends and forms of representation used on the note to the shopkeeper among Swedish children

The results of examining Swedish participants' use of representation on the Milk Note across the age-groups is shown in table 4.24.

Table 4.24 Relationship between form of representation used on the Milk Note and age among Swedish children

Age groups (age in months)	N	Blank	Idio	Pic	Ico	Sym
1: 36-41	8	0	2	2	4	0
2: 42-49	17	0	4	3	8	2
3: 50-54	10	0	4	1	4	1
4: 55-60	13	0	4	1	3	5

These results show that no child left the note blank and that the use of idiosyncratic remained about the same while the figures for pictographic were very low. Both pictographic and iconic show a slight drop with age, whereas the table suggests there was some increase in the use of symbolic among the very oldest Swedish children.

The only reliable association was between age and use of the symbolic form of representation on the note for the milkman and the decline in use of iconic approached significance (Iconic, $r_s = -.24$, $p < .1$; Symbolic, $r_s = .34$, $p < .025$; Idiosyncratic, $r_s = .06$, ns; Pictographic, $r_s = -.11$, ns)

Comparison across countries

Table 4.25 allows for comparison of results for the Milk Note task in the three countries using the two comparable age groups 2 and 4. It can be seen that whereas none of the English or Swedish participants left the note blank, 2 out of 10 younger children in Japan did so and 2 out of 22 in the older group.

Table 4.25 Comparison of forms of representation used on the Milk Note across countries according to age-group

Age groups	N	Blank	Idio	Pic	Ic	Sym
2: (42-49 months)						
England	24	0	12	1	8	3
Japan	10	2	4	2	0	2
Sweden	17	0	4	3	8	2
4: (55-60 months)						
England	24	0	11	1	5	7
Japan	22	2	6	6	1	7
Sweden	13	0	4	1	3	5
Totals:						
England	48	0	23	2	13	10
Japan	32	4	10	8	1	9
Sweden	30	0	8	4	11	7

Amongst the younger children, the only type of representation whose use varied with country was iconic, according to likelihood ratio chi-square tests (Idiosyncratic, chi-square = 3.03, df = 2, ns; Symbolic, chi-square =0.39, df =2, ns; Pictographic, chi-square = 2.78, df = 2, ns; Iconic, chi-square = 9.39, df =2, $p < .01$). This was largely due to hardly any of the Japanese children using the iconic form of representation (England v Japan, U = 80, $p < .05$; England v Sweden, U = 176, ns; Sweden v Japan, U = 45, $p <.05$).

Amongst the older children, type of representation did not vary reliably between countries though variation in use of pictographic representation approaches significance with older Japanese children using it a little more often than in England and Sweden (Idiosyncratic, chi-square = 1.89, df = 2, ns; Symbolic, chi-square = 0.33, df =2, ns; Pictographic, chi-square = 5.69, df = 2, $p<.1$; Iconic, chi-square = 3.65, df =2, ns).

These findings, obtained by comparing the results on the note to the milkman or shopkeeper with those on the Tins Game, appear to discount the theory that children might find the Milk Note task more meaningful - as all groups of participants used less, rather than more, sophisticated ways of representing cardinal number than they had on the tins.

4.2.1 Overview of results of the Milk Note task

The task of writing a note to the milkman or shopkeeper was designed to create a context that was more familiar to participants when asked to represent cardinality. As the note would be read by somebody else, not just by the child, the task was chosen to examine whether this context might elicit a more mature and hence more successful response. The results show that overall, performance actually declined rather than improved on the Milk Note. Whereas hardly any children left the label blank on the tin with 2 blocks, on the Milk Note more older than younger children left the paper blank when asking for two bottle or cartons of milk.

Results were similar across countries, the only significant difference was that almost no Japanese children used an idiosyncratic from of representation on the Milk Note. It is interesting to note that Japanese children also made least use of idiosyncratic in the Tins Game. This may suggest a cultural difference in that Japanese children are less likely to invent their own symbols generally. Furthermore, they were almost all successful in communicating 'two' either by using symbolic representation or by devising a transparent way of capturing one-to-one correspondence through using pictographic or iconic forms of representation. In short, performance was generally less successful on the Milk Note than in the tins game, therefore the task was clearly less effective in eliciting the use of number symbols.

It is interesting to note that prior to the study only the English participants had previously been exposed to the cultural practices of a milkman delivering milk and of customers writing a note to state the number of bottles required. For the purpose of this experiment, it is useful to compare the results of the English cohort on the milk note task with results in Japan in Sweden where the practice of delivering and ordering milk does not exist. When viewed in this way it is interesting to note that the results of the English children were statistically no better than in Japan and Sweden.

The overall results of the milk note task suggest that the act of communicating cardinal number to another person is unfamiliar to all young children and is even *less* meaningful than communicating or representing number to themselves for their own purposes. As the use of symbolic representation actually fell on the milk note, it suggests that young children do not recognise numerals as an accepted way of communicating cardinal number within society.

On the Milk Note this may be largely due to metacognitive factors; more older than younger children left the paper blank, which suggests that the older children may have declined to commit themselves to paper due to awareness of their uncertainty about how to represent 2 units of milk in this particular social setting.

4.3 Oral representation of cardinal number in the Fast Food task

This test required participants to speak on a toy telephone to place an order for two, one and three fast-food items. Their speech was noted on a schedule and their responses subsequently allocated one of six codes according to which best described their use of number, singularity or plurality, and their naming of the items.

The coding process then enabled the children to be placed in a category according to their performance across the three trials as either:

1) correct specification of object and quantity using number words

2) correct specification of objects and quantity using a combination of number words and a 'listing' of quantity (e.g. "a burger and a burger" for 2 burgers)

3) correct specification of objects and quantity using listing only

4) correct specification of number words, but omitting any mention of objects

5) correct specification of objects but no specification of quantity

6) other responses

Table 4.26 Overall frequencies of codings according to age-group. N=167

Age groups (age in months)	N	Fast-food codings					
		1	2	3	4	5	6
1: 36-41	8	0	0	0	0	0	8
2: 42-49	51	4	3	0	0	9	35
3: 50-54	44	2	1	0	1	27	13
4: 55-60	59	5	3	0	0	24	27
5: 61-62	5	0	0	0	0	4	1
Totals:	167	11	7	0	1	64	84

To examine whether there was any relation between responses on the fast-food order and age, the fast-food orders were re-coded as either successful (codes 1, 2, 3) or unsuccessful (4, 5, and 6). A X^2 analysis showed no significant variation with age (X^2 = 3.22, df = 4, ns). However, a further inspection of data for the different countries indicated that something unusual had happened.

Table 4.27 Overall frequencies of codings according to country

Country	N=	Fast-food codes					
		1	2	3	4	5	6
England	48	8	6	0	0	7	27
Japan	71	3	1	1	1	57	9
Sweden	48	0	0	0	0	0	48

None of the Swedish children, and only a very small number of Japanese children, gave orders that specified both quantity and object type. This suggests that the task may have been poorly understood by them, either because of deficiencies in administration of the task or because the context of playing at ordering food on the telephone made little sense to them. Even amongst the English children however there was no relation between successful ordering and age (X^2 = 0, df=1, ns).

Amongst the English children, it was found that successful ordering was related to performance on the Tins game: children who used idiosyncratic representation more often were less likely to succeed in the fast-food task (r_s = -.33, $p < .05$).

4.3.1 Overview of results of the Fast Food task

The Fast Food task was designed to further extend elements of the tins game and milk note tasks; children's understanding of cardinal number was again explored in a setting that was familiar but even more meaningful to the children. Responses did not rely upon either a knowledge of numerals or on children's invention of a form of graphical representation but solely on their spontaneous expression using 'everyday' speech. One, two and three items were included in order to explore the children's use of the same range of simple numbers as in the tins game and milk note task.

The findings show that on the Fast Food Order very few participants overall used cardinal number orally. None of the Swedish children, very few Japanese and only a small number of English children were successful. There could be several reasons.

Firstly, it could be that the task was poorly understood by participants due to the way it was explained and administered. However, the task itself was perhaps no more 'novel' than any of the other tasks, none of which were routine for the 3-5 year-olds. Therefore it could be argued that if the children had grasped the adults' explanations of the other tasks, they should also have been able to understand what to do for the Fast Food Order.

Secondly, it could be that young children are not used to open-ended questions concerning cardinal number which require them to respond orally. Questions such as 'how many?' may be asked in the school context by referring to familiar items used for counting purposes such as blocks or counters. They may also be strategically arranged for the purpose or pointed to in a counting book. In such situations the child provides an answer *in response* to a direct question, which was not the case in the study.

Conversely, in the Fast Food task the child was not directly asked to respond to 'how many?' but required to integrate the information into spontaneous speech. This form of response is semantically different and more complex than a simple one-word answer (such as 'two'), and requires the child to incorporate a cardinal number into the structure of his sentence.

Although the task may be criticised because the responses required appear complex, it was precisely the nature of participants' responses that the task was designed to investigate. Furthermore, the children's responses were examined for all the ways in which they might represent cardinality in an oral way. The examination found that even the use of a cardinal number on its own, without mention of the food item (e.g. "three"), was very low, and that the use of one-to-one correspondence through listing (e.g. "a burger and a burger"), which could be seen as similar to pictographic and iconic forms of graphical representation, was not used at all. Therefore neither a complex nor prescribed format were necessary, as a range of responses were acceptable.

Thirdly, it could be argued that the situation was not seen by participants as realistic, involving a toy telephone and only photographs of items, so perhaps they did not fully engage with the task. However, all children responded to the prompts, participated fully and the majority included the *name* of the food items in their speech when looking at each photo; in just a few cases the wrong items were named.

Because levels of participation were high, and as children's responses were generally 'appropriate' to what they were asked, it seems likely that the task was not poorly understood.

When comparing how participants represented 'two items' on the fast food order with the way they represented 'two blocks' and 'two bottle/cartons of milk' on the tins game and milk note, the pattern of responses could be seen as related.

Table 4.28 below compares the use of cardinality, either in spoken form or as a symbol, with graphical and spoken forms of representation that rely on one-to-one correspondence such as pictographic and iconic markings and oral listing.

Table 4.28: A comparison of ways of representing 2 as a cardinal number in graphical and oral form N=167

Task	One-to-one	Cardinal	Other/idiosyncratic
Tins game	73	62	32
Milk note	61	40	66
Fast food	0	19	148
Total	134	121	246

It is important to note that results on the tasks which involved *cardinal* aspects of number became progressively lower (i.e tins game, milk note and fast food). Furthermore, this decrease in the use of cardinal number correlates with the degree to which the child was required to communicate cardinality to another person. The ability to count, write a numeral or give a cardinal number in response to a direct 'how many?', are not in question. What appears to be the case is that there is some difficulty relating to the children's knowledge of how and when number is used in society for communicating cardinality, whereas in general their use of number for other purposes was relatively further developed, or at least a developmental delay.

The difficulties that these young children experienced when communicating cardinality to another person could be attributed to egocentrism, to a universal stage in cognitive development or to socio-cultural experiences which excluded a demonstration of the relevance and power of numerals to communicate cardinal aspects of number.

This hypothesis clearly requires further investigation, however if the communication theory is substantiated it would provide important information on the degree to which young children understand that written numerals and the use of numbers in everyday speech are current and powerful ways of communicating cardinal number.

For ease of reference and comparison, the following summary charts showing the most significant findings of the three studies have been provided.

Table 4.29 Summary of results of the Tins Game

Participants	Aspect of performance	Significant findings	Changes with age
Overall	**Representing tin with 0 blocks**	Used Idiosyncratic, symbolic or left label blank	None
Overall	**Representing tins with 1, 2 and 3 blocks**	Representation similar on all three tins:	Pictographic and Blank no change
		Use of Iconic and Symbolic (mutually similar) Idiosyncratic and Pictographic (mutually similar)	Less use of Idiosyncratic and Iconic, and more use of Symbolic with age
England	Representing tin with 0 blocks	A quarter of participants left label blank.	No change with age.
	Representing tins with 1, 2 and 3 blocks	Idiosyncratic varied somewhat according to number of blocks. Pictographic varied according to numbers of blocks.	Decline in Idiosyncratic with age. No change with age in use of Pictographic or Iconic. Increase in Symbolic use with age.
Japan	Representing tin with 0 blocks	A third of participants left label blank.	No change with age.
	Representing tins with 1, 2 and 3 blocks	No significant difference between tins.	Decline in Idiosyncratic with age. No change with age in use of Pictographic or Iconic. Increase in Symbolic use with age.
Sweden	Representing tin with 0 blocks	No significant trend in leaving label blank.	
	Representing tins with 1, 2 and 3 blocks	Most used Idiosyncratic. No significant difference between tins.	No changes in Idiosyncratic or Pictographic with age. Increase in Iconic and Symbolic with age.

Participants	Aspect of performance	Significant findings	Changes with age
Comparing performance in England Japan and Sweden	**Representation on Tins 1, 2, and 3**	Across the *younger age-groups*: Japanese participants made least use of Idiosyncratic and most use of Symbolic. Swedish made more use of Iconic . No differences in leaving label blank or use of Pictographic No difference when comparing *older age groups.*	
Overall	**Accuracy in recalling numbers of blocks** when reading back their labels, according to type of representation used	Idiosyncratic was 33% successful Leaving label blank was 90% successful Pictographic, Iconic, Symbolic 90% successful	Improvement in accuracy with age, but mainly due to changes in the type of representation used

Table 4.30 Summary of results of the Note to the Milkman or Shopkeeper (Milk Note)

Participants	Aspect of performance	Significant findings	Changes with age
Overall	Comparing type of representation used for 2 units of milk on the **Note to Milkman or Shopkeeper** with representation used on the Tin with 2 blocks	Use of Idiosyncratic increased on Milk Note Pictographic was the same as on Tin 2 Iconic and Symbolic fell by a third	*Older children* were less likely to use Iconic and Symbolic on the Milk Note, and more likely to leave the note Blank or use Idiosyncratic
Comparing performance in England Japan and Sweden	**Type of representation used on the Milk Note**	Japanese children (of all ages) made almost no use of Idiosyncratic	

Table 4.31 Summary of results of the Fast Food Order

Participants	Aspect of performance	Significant findings	Changes with age
Overall	**Use of number orally on the Fast Food Order (1-3)**	Few participants used number: a quarter of English, few Japanese, no Swedish used number.	No improvement with age.
	Relationship between oral use of number on Fast Food Task and visual representation in the Tins Game	English children who were less successful on the Fast Food Order were those more likely to use Idiosyncratic in the Tins game.	

Chapter 5

Results of tests involving nominal, cardinal and ordinal uses of number: the Magic Book

Results for this set of tests are presented according to the three stages in which they were administered. Firstly, participants were asked to say whether they thought anything was missing from the item in the picture and what that was; secondly they were given a drawn picture of the same item and asked to "put in what's missing"; thirdly they were shown a photograph of the item on which the missing numeral was in its usual place and asked "Why is the number there?".

Participants in England, Sweden and Japan were all shown the same two items, a coin and a telephone. In addition to these, participants in England and Sweden were shown a bus and a birthday card. In Japan participants were shown a tv remote control unit in addition to the coin and telephone.

Table 5.1 Summary of items used in each of the countries

	Coin	Phone	Card	Bus	TV control
England	X	X	X	X	-
Japan	X	X	-	-	X
Sweden	X	X	X	X	-
Total N=	167	167	96	96	71

5.1 "What is missing?"

In this test, participants were shown pictures of items which had numerals missing and asked what they thought was missing from the item. Table 5.2 shows the overall results for the two items that were shown to participants in all three countries.

Table 5.2 Overall frequencies of responses to "What is missing from the coin and the phone?"

Item	N	Number	Other
Coin	167	35	132
Phone	167	67	100
Bus	96	47	49
Card	96	58	38
TV	71	8	63

As table 5.2 indicates, participants found it more difficult to identify what was missing from the coin, answered correctly by only a fifth. Less than a half were able to identify a number as missing from the telephone.

Relationship between results for "What's missing?" and age

Results are examined within each country according to age-group, using the same age bands used in the other studies and as shown in table 5.3

Table 5.3 Participants' age groups in the different countries

Age group	Country	N	Age range (months)	Mean age (months)
1	Sweden	8	36 - 41	39
2	England	24	43 - 49	46
	Japan	10	42 - 49	46
	Sweden	17	42 - 49	45
3	Japan	34	50 - 54	52
	Sweden	10	50 - 54	52
4	England	24	55 - 58	57
	Japan	22	55 - 60	57
	Sweden	13	55 - 58	56
5	Japan	5	61 - 62	61

Separate countries

England

Results for "What's missing?" were examined among the English cohort according to item.

Table 5.4 Overall frequencies of responses to "What's missing?" by English participants N=48

Item	N	Number	Other
Coin	48	15	33
Phone	48	32	16
Bus	48	28	20
Card	48	27	21

Table 5.4 shows different results on all four items in England. According to Cochran Q tests, and follow-up McNemar tests, English participants were significantly less likely to identify the coin as having a numeral missing than the other items ($Q= 19.32$, $df=3$ $p = .000$).

Relationship between results for "What's missing?" and age

Results were examined according to each age-group for each of the items.

Table 5.5 Relationship between "What's missing from the coin?" and age among English participants N=48

Age groups (age in months)	N	Number	Other
2: 42-49	24	8	16
4: 55-60	24	7	17

The English results for the coin are shown in table 5.5 and appear to be at a generally much lower success rate with no improvement with age. Chi-Square tests confirm no significant improvement with age on responses to the coin ($X^2 = .097$ $df =1$, $p = .755$ ns).

Table 5.6 Relationship between responses to "What's missing from the phone?" and age among English participants N=48

Age groups (age in months)	N	Number	Other
2: 42-49	24	13	11
4: 55-60	24	19	5

Results for "What's missing from the phone?" among English children, shown in table 5.6 , appear a little better, particularly among the older children. Chi-Square tests confirm an improvement with age for responses to the phone which approaches significance ($X^2(2)$= 3.438 df =1, p = .064).

Table 5.7 Relationship between responses to "What's missing from the bus?" and age among English participants N= 48

Age groups (age in months)	N	Number	Other
2: 42-49	24	12	12
4: 55-60	24	16	8

Table 5.7 shows results for the bus for the two English age groups which suggest a small improvement with age. However, a comparison of the results between younger and older children using likelihood ratio Chi-Square tests found no significant improvement with age for responses to the bus (X^2 = 1.379 df =1, p = .240 ns).

Table 5.8 Relationship between responses to "What's missing from the birthday card ?" and age among English participants N=48

Age groups (age in months)	N	Number	Other
2: 42-49	24	10	14
4: 55-60	24	17	7

Results for the birthday card are shown in table 5.8 and again show some improvement with age. Chi-Square tests showed that among the English children there was a significant improvement with age for responses to the birthday card (X^2 = 4.214 df =1, p = .04).

Japan

Results for responses to "What's missing?" for the Japanese participants are shown in the following tables.

Table 5.9 Overall responses among Japanese children to "What's missing?" N = 71

Item	N	Number	Other
Coin	71	4	67
Phone	71	13	58
TV Control	71	8	63

Successful responses to all three items were low, with only 13 children able to note that a number was missing from the phone, which was the best response at under a fifth of all participants. According to Cochran Q tests Japanese children were more likely to identify a numeral as missing from the phone and less likely to do so on the coin (Q = 9.385 df=2, p = .009), follow-up McNemar tests confirming this (p = .012).

Relationship between results for "What's missing?" and age

Results for each item were then examined according to age-group.

Table 5.10 Relationship between responses to "What's missing from the coin?" and age among Japanese participants

Age groups (age in months)	N	Number	
2: 42-49	10	0	10
3: 50-54	34	3	31
4: 55-60	22	1	21
5: 61-62	5	0	5

Results for the coin, shown in table 5.10 are consistently very low or at zero. Numbers were too small to justify following up with statistical tests.

Table 5.11 Relationship between responses to "What's missing from the phone ?" and age among Japanese participants N=71

Age groups (age in months)	N	Number	Other
2: 42-49	10	0	10
3: 50-54	34	9	25
4: 55-60	22	4	18
5: 61-62	5	0	5

Table 5.11 shows Japanese children's responses to the phone according to age-group, which were generally low with no improvement in the older groups. Again, numbers of children who identified that a number was missing were too small to make statistical tests worth considering.

Table 5.12 Relationship between responses to "What's missing from the TV control?"and age among Japanese participants N=96

Age groups (age in months)	N	Number	Other
2: 42-49	10	0	10
3: 50-54	34	5	29
4: 55-60	22	3	19
5: 61-62	5	0	5

Results in table 5.12 show the results for the tv control to be consistently low or at zero across all age groups. Numbers are too small to justify statistical analyses.

Sweden

Results for Swedish children are shown in table 5.13 which sets out success rates when asked "What's missing?" according to item.

Table 5.13 Overall results among Swedish participants to "What's missing?" N = 48

Item	N	Number	Other
Coin	48	16	32
Phone	48	22	26
Bus	48	19	29
Card	48	31	17

The coin appears to be the most difficult followed by the bus, and the phone at around half of all participants. The birthday card seems to have been the easiest to identify. Cochran Q results show that Swedish children were significantly more likely to identify the birthday card as having a number missing than the other items (Q= 19.86, df = 3 p = .000), with McNemar tests confirming this (p = .004).

Relationship between results for "What's missing?" and age.

Results were then examined according to age-group.

Table 5.14 Relationship between responses to "What's missing from the coin ?" and age among Swedish participants N = 48

Age groups (age in months)	N	Number	Other
1: 36-41	8	2	6
2: 42-49	17	8	9
3: 50-54	10	2	8
4: 55-60	13	4	9

Results among Swedish children for the coin are shown in table 5.14, with a mixed picture of between a quarter and a half of children successful across the age-groups. Chi-Square tests confirmed that there was no significant variation with age for responses to the coin (X^2 = 2.544 df = 3, p = .467 ns).

Table 5.15 Relationship between responses to "What's missing from the phone ?" and age among Swedish participants N = 48

Age groups (age in months)	N	Number	Other
1: 36-41	8	4	4
2: 42-49	17	6	11
3: 50-54	10	4	6
4: 55-60	13	8	5

Table 5.15 shows results for the phone for Swedish children where responses appear quite good, with response rates consistently at around fifty per cent across all age-groups. Chi-Square tests showed that there was no significant variation with age for responses to the phone (X^2 = 2.26 df = 3, p = .52 ns).

Table 5.16 below shows success rates among Swedish children to "What's missing from the bus?" according to age:

Table 5.16 Relationship between responses to "What's missing from the bus?" and age among Swedish participants N = 48

Age groups (age in months)	N	Number	Other
1: 36-41	8	2	6
2: 42-49	17	8	9
3: 50-54	10	3	7
4: 55-60	13	6	7

As the table shows, between a quarter and a half of children in each age-group responded successfully. A comparison of the results among younger and older Swedish children using likelihood ratio Chi-Square tests found no significant variation with age for responses to the bus($X^2 = 1.776$ df = 3, p = .62 ns).

Table 5.17 Relationship between responses to "What's missing from the card ?" and age among Swedish participants N = 48

Age groups (age in months)	N	Number	Other
1: 36-41	8	6	2
2: 42-49	17	13	4
3: 50-54	10	4	6
4: 55-60	13	8	5

Table 5.17 above appears to suggest a similar pattern for the card and Chi-Square tests confirmed that there was no significant variation with age for responses to the birthday card ($X^2 = 4.068$ df = 3, p = .254 ns).

Comparison across countries

Comparisons of results for "What's missing?" were made between countries, using results for the two common age-groups 2 and 4, as shown below.

Table 5.18 Frequencies of responses to "What's missing from the coin?" among participants in England, Japan and Sweden N=167

Age groups (age in months)	N	Number	Other
2: 42-49	51		
England	24	8	16
Japan	10	0	10
Sweden	17	8	9
4: 55-60	59		
England	24	7	17
Japan	22	1	21
Sweden	13	4	9

Table 5.18 shows that none of the Japanese were able to identify a numeral as missing from the coin, compared to a third of English and a half of Swedish participants. There was little or no improvement in England and Japan, and in Sweden the rate fell with age.

The frequencies of younger children stating that the numeral was missing from the coin varied with country ($X^2 = 6.56$, $df = 2$, $p < .05$). This was due to none of the Japanese children pointing this out. There was no significant difference between English and Swedish children ($X^2 = 0.78$, $df= 1$, ns).

In the older age-group the frequencies differed with country to some extent ($X^2 = 5.41$, $df = 2$, $p = .067$). Again this seemed largely due to the Japanese children being less likely to point out that numbers were missing from the coin as the proportions of English and Swedish children were almost identical ($X^2 = 0.01$, $df = 1$, ns).

Table 5.19 Frequencies of responses to "What's missing from the phone?" among participants in England, Japan and Sweden N=167

Age groups (age in months)	N	Number	Other
2: 42-49	51		
England	24	13	11
Japan	10	0	10
Sweden	17	6	11
4: 55-60	59		
England	24	19	5
Japan	22	4	18
Sweden	13	8	5

In table 5.19 we can see that in England around half of the younger group were able to say what was missing from the phone, around a third in Sweden, but no children in Japan.

Success rates improved with age in all three countries. Japanese children improved from zero to about a fifth, the Swedish from a third to almost two-thirds in the older age-group, and English children from a half to three-quarters.

Chi Square tests confirmed that in the younger age-group frequencies differed on the phone, ($X^2 = 8.9$, df = 2, $p < .02$). This was largely due to the Japanese children as there was no difference between the English and Swedish ($X^2 = 1.4$, df = 1, ns).

Among the older age-group, frequencies on the phone did differ between countries ($X^2 = 17.7$, df = 2, $p = .000$) which was again due to the performance of the Japanese participants. There was no difference between that of the English and Swedish children ($X^2 = 1.3$, df = 1, ns).

Table 5.20 Frequencies of responses to "What's missing from the bus?" among participants in England and Sweden N=78

Age groups (age in months)	N	Number	Other
2: 42-49	41		
England	24	12	12
Sweden	17	8	9
4: 55-60	37		
England	24	16	8
Sweden	13	6	7

Results for England and Sweden are shown for the bus in table 5.20 above. These show that in both England and in Sweden half of the younger children were able to identify a number as missing from the bus. In England this improved to two thirds in the older age-group and in Sweden to about half.

Among the younger children Chi Square tests confirmed that there was no difference between English and Swedish children in pointing out that there was a numeral missing from the bus ($X^2 = 0.0$, df = 1, ns) Among the older group no difference was found between English and Swedish participants ($X^2 = 1.5$, df = 1, ns).

Table 5.21 Frequencies of responses to "What's missing from the birthday card?" among participants in England and Sweden N=78

Age groups (age in months)	N	Number	Other
2: 42-49	41		
England	24	10	14
Sweden	17	13	4
4: 55-60	37		
England	24	17	7
Sweden	13	8	5

In table 5.21 results for the birthday card show that just under half of the younger group in England were successful compared to three quarters of Swedish children. However the picture changes in the older age-groups as English participants' performance improved from a half to nearly three-quarters, close to Swedish participants at about two-thirds.

Among the younger age group Chi Square tests showed that Swedish children were more likely to point out that there was a numeral missing from the card (X^2 = 4.9, df = 1, p< .05), whereas there was no difference in the older group between English and Swedish (X^2 = 0.3, df = 1, ns).

5.2 "Put in what's missing"

On being asked to 'put in' what was missing, participants' responses were then analysed according what they entered on the corresponding pictures of the items.

Table 5.22 Overall frequencies of responses to "Put in what's missing?" N=167

Item	N	Number	Other
Coin	167	42	125
Phone	167	72	95
Bus	96	30	66
Card	96	30	66
TV control	71	27	44

On the two items that were common to all three countries, there was little difference between performance on the coin and the phone, both at which were around a quarter of all participants. The bus and card were similar and a little better at just under a third, with the tv control the best at over three-quarters.

The overall relationship between what was entered and age was examined within each country.

Separate countries

Results for what was entered as missing from each item were then examined for each country.

England

Table 5.23 Overall frequencies of responses to "Put in what's missing" by English participants N=48

Item	N	Number	Other
Coin	48	11	37
Phone	48	23	25
Bus	48	18	30
Card	48	18	30

Table 5.23 shows the overall results according to item among English children. According to Cochran Q tests and follow-up McNemar tests, the coin was significantly less likely to elicit a correct response ($Q = 9.955$ df=3, $p = .019$) than the phone.

Relationship between what was entered and age

Table 5.24 Relationship between responses to "Put in what's missing from the coin" and age among English participants N=48

Age groups (age in months)	N	Number	Other
2: 42-49	24	5	19
4: 55-60	24	6	18

On the coin, where correct responses were low overall, there was no significant improvement with age ($X^2(2)$= .118 df = 1, p = .731 ns).

Table 5.25 Relationship between responses to "Put in what's missing from the phone" and age among English participants N=48

Age groups (age in months)	N	Number	Other
2: 42-49	24	5	19
4: 55-60	24	18	6

Tests show that on the phone the older children were significantly more likely to put in a numeral ($X^2(2)$= 14.903 df = 1, p = .000).

Table 5.26 Relationship between responses to "Put in what's missing from the bus " and age among English participants N=48

Age groups (age in months)	N	Number	Other
2: 42-49	24	4	20
4: 55-60	24	14	10

Results for what was entered on the bus by English children are shown in table 5.26. Responses by participants were compared using likelihood ratio Chi-Square tests which showed that a numeral was significantly more likely to be entered on the bus by older children ($X^2(2)$= 9.282 df = 1, p = .002).

Table 5.27 Relationship between responses to "Put in what's missing from the birthday card" and age among English participants N=48

Age groups (age in months)	N	Number	Other
2: 42-49	24	4	20
4: 55-60	24	14	10

On the birthday card there was a significant improvement with age, with older children more likely to enter a numeral ($X^2(2)$= 9.28 df = 1, p = .002).

Japan

Results for what was entered by Japanese participants are a also considered according to item.

Table 5.28 Overall frequencies of responses to "Put in what's missing" by Japanese children N = 71

Item	N	Number	Other
Phone	71	33	38
Coin	71	23	48
TV control	71	27	44

Results for the three items shown to the Japanese children can be seen in table 5.28 These suggest that the phone was found easiest and the coin the most difficult. Cochran Q and follow-up McNemar tests confirm that participants were less likely to successfully enter what was missing on the coin than on the phone (Q = 7.328 df=2 p = .027).

Relationship between results for what was entered and age

Results for Japanese children were then examined according to item for each age-group in order to identify whether there was any improvement with age.

Table 5.29 Relationship between responses to "Put in what's missing from the coin" and age among Japanese participants N=71

Age groups (age in months)	N	Number	Other
2: 42-49	10	4	6
3: 50-54	34	10	24
4: 55-60	22	5	17
5: 61-62	5	4	1

Table 5.29 shows results for the coin, with around a third to a half of children successful in age-groups 2, 3 and 4, as well as 4 out of 5 children in the smaller, oldest age-group. McNemar test show that there was no real difference between the age-groups, and Chi-Square tests confirm no

significant improvement with age on the coin ($X^2(_2)$= 6.192 df = 3, p = .103 ns).

Table 5.30 Relationship between responses to "Put in what's missing from the phone " and age among Japanese participants N=71

Age groups (age in months)	N	Number	Other
2: 42-49	10	5	5
3: 50-54	34	13	21
4: 55-60	22	10	12
5: 61-62	5	5	0

Results for Japanese children on what was entered on the telephone can be seen in table 5.30. These show a difference between the success rate of under a half in groups 2 and 3, and over a half in the two oldest age-groups. Chi-Square tests confirm that there was a significant improvement with age on the telephone ($X^2(_2)$= 8.661 df = 3, p = .034).

Table 5.31 Relationship between responses to "Put in what's missing from the TV control" and age among Japanese participants N=71

Age groups (age in months)	N	Number	Other
2: 42-49	10	4	6
3: 50-54	34	12	22
4: 55-60	22	7	15
5: 61-62	5	4	1

Results for the tv control are shown in table 5.31 above, which indicates that about a third in age groups 2, 3 and 4 were successful, and 4 out of the 5 children in the small group of older children.

Chi-Square tests confirm no significant improvement with age in the use of numerals on the tv control ($X^2(_2)$= 4.182 df = 2, p = .24 ns).

Sweden

Results for what was entered by Swedish children was firstly examined according to item.

Table 5.32 Overall frequencies of responses to "Put in what's missing"by Swedish participants N=48

Item	Number	Other
Coin	8	40
Phone	16	32
Bus	12	36
Card	12	36

Table 5.32 above suggests that overall the telephone was found easiest and the coin more difficult by Swedish participants. However Cochran Q tests show that this was not significant(Q=5.818 df=3, p =.121 ns).

Relationship between results for what was entered and age

Results for Swedish children were also examined for each item according age-group to identify any improvement with age.

Table 5.33 Relationship between responses to "Put in what's missing from the coin" and age among Swedish participants N = 48

Age groups (age in months)	N	Number	Other
1: 36-41	8	0	8
2: 42-49	17	3	14
3: 50-54	10	1	9
4: 55-60	13	4	9

The responses by Swedish children to what was entered on the coin appear to be consistently low in all age-groups. Chi-Square tests confirmed no significant changes with age in responses to the coin ($X^2(_2)$= 4.86 df = 3, p = .182 ns).

Table 5.34 Relationship between responses to "Put in what's missing from the phone" and age among Swedish participants N = 48

Age groups (age in months)	N	Number	Other
1: 36-41	8	1	7
2: 42-49	17	4	13
3: 50-54	10	3	7
4: 55-60	13	8	5

The results for the phone, shown in table 5.34, show that around a quarter of children were successful in age-groups 2 and 3, rising to over a half in the oldest group. Chi-Square tests detected that the changes with age in responses to the telephone were at a level which approaches significance ($X^2(2)$= 6.986 df =3, p = .072).

Table 5.35 Relationship between responses to "Put in what's missing from the bus" and age among Swedish participants N = 48

Age groups (age in months)	N	Number	Other
1: 36-41	8	1	7
2: 42-49	17	4	13
3: 50-54	10	2	8
4: 55-60	13	5	8

Table 5.35 shows that apart from the younger children, the results for the bus remain consistently at around a fifth to a third in all other age-groups. Likelihood ratio Chi-Square tests detected no significant changes with age in responses to what was entered on the bus ($X^2(2)$= 2.074 df = 3, p = .557).

Table 5.36 Relationship between responses to "Put in what's missing from the card" and age among Swedish participants N = 48

Age groups (age in months)	N	Number	
1: 36-41	8	1	7
2: 42-49	17	5	12
3: 50-54	10	2	8
4: 55-60	13	4	9

Responses for the birthday card, shown in table 5.36 are low. Likelihood ratio Chi-Square tests confirmed no significant changes with age in responses to the birthday card ($X^2(2)$= 1.302 df = 3, p = .729).

Comparison across countries

An overall comparison of what children entered was carried out using the two common age-groups. All three countries were compared on the two common items of the coin and the telephone.

Table 5.37 Frequencies of responses to "Put in what's missing from the coin" according to country and age N=110

Age groups (age in months)	N	Number	
2: 42-49	51		
England	24	5	19
Japan	10	4	6
Sweden	17	3	14
4: 55-60	59		
England	24	6	18
Japan	22	5	17
Sweden	13	4	9

Table 5.37 shows that in the younger age-group only around a sixth of younger children in Sweden and around a fifth of those in England were able to put in what was missing from the coin; just under a half were able to do so in Japan.

Even among the older age-group the majority of children were unable to put in what was missing from the coin; in England a quarter did so and in Japan the figure fell to just over a quarter, whereas in Sweden it rose to a third - but the numbers involved were very small.

Likelihood ratio Chi-Square tests found no significant variation between the countries in what was entered in response to the coin among younger children ($X^2(_2)$= 1.783 df=2, p = .410 ns) or older children ($X^2(_2)$= .277 df=2, p = .871 ns).

Table 5.38 Frequencies of responses to "Put in what's missing from the phone" according to country and age N=110

Age groups (age in months)	N	Number	Other
2: 42-49	51		
England	24	5	19
Japan	10	5	5
Sweden	17	4	13
4: 55-60	59		
England	24	18	6
Japan	22	10	12
Sweden	13	8	5

On the other common item, the telephone, success rates appear consistently lower among the younger children in all countries; a fifth of English children were successful, compared to a quarter of the Swedish, and the Japanese children were the most successful as a half were able to put in numerals as missing from the phone.

There was an improvement with age in England and Sweden; older English children scored best with three-quarters successful, and Swedish children achieved well over half, whereas Japanese participant's scores remained about the same at a half.

On the phone, likelihood ratio Chi-Square tests detected no significant difference between them in responses by younger children in all three countries ($X^2{}_{(2)}$= 2.968 df=2, p =.227) and older participants ($X^2{}_{(2)}$= 4.272 df=2, p = .118 ns).

The performance of participants in England and Sweden was compared on two other items, the bus and the birthday card.

Table 5.39 Frequencies of responses to "Put in what's missing from the bus" according to country and age N= 78

Age groups (age in months)	N	Number	Other
2: 42-49	41		
England	24	4	20
Sweden	17	4	13
4: 55-60	37		
England	24	14	10
Sweden	13	5	8

In table 5.39 it can be seen that only a sixth of younger children in England and a quarter in Sweden were able to put in numerals as missing from the bus. Looking at the older age-group, in Sweden the figure rose to over a third and in England to over a half.

Likelihood ratio tests show that the entry of numerals did not vary for either age-group (younger, ($X^2(_1)$= 0.3, ns ; older, ($X^2(_1)$= 1.34, ns).

Table 5.40 Frequencies of responses to "Put in what's missing from the birthday card" according to country and age N=78

Age groups (age in months)	N	Number	Other
2: 42-49	41		
England	24	4	20
Sweden	17	5	12
4: 55-60	37		
England	24	14	10
Sweden	13	4	9

On the birthday card, table 5.40 shows that only a sixth of younger English participants and less than a third of Swedish entered a numeral as missing. In both countries performance improved with age; over a half of older children in England entered a numeral and in Sweden more than a third, but likelihood ratio tests reveal that the entry of numerals did not vary significantly between countries for either age group (younger ($X^2(_1)$= 0.93, ns; older, ($X^2(_1)$= 2.62, ns).

5.3 "Why is the number there?"

Next, participants were shown the Magic Book containing pictures of the same set of items as before but with the numerals displayed in their usual positions. They were then asked "Why is the number there?". If the child's reply clearly indicated that she understood its purpose it was classified as successful; for example on the telephone it was considered a successful response if the child replied: "you've got to press the right numbers if you want to phone the person". If the child made only general reference to the numbers it was classified as an incorrect response, for example "you must press the numbers". On the coin it was considered a successful response if the child explained the coin "It's a 2 because it's 2p (pence) and you can buy something for 2p". It was not considered to be successful if the child said "That's a 2. It's money".

For the bus, a response was deemed as successful if the child explained in some way that the number was related to the route, for example "If you get the twenty-five you can go to the shops", but not if this was omitted such as "Cos it's a twenty-five." Similarly a successful explanation of the birthday card would refer to age, such as "That's four cos he's four; it's his birthday", but not "That's a four on his card". On the television control unit a successful response would refer to the channel numbers, such as "If you press the numbers you can get the programme, like on five I watch cartoons I do", but unsuccessful if the child said "You press the numbers. It's for the telly".

Overall frequencies to being asked to give reasons for a number on the items can be seen below in table 5.41.

Table 5.41 Overall frequencies of responses to "Why are there numbers there?" according to item

Item	N	Explained	Not explained
Coin	167	35	132
Phone	167	67	100
Bus	96	49	47
Card	96	58	38
TV control	71	23	48

These results suggest that the number on the birthday card was the easiest to explain as almost two thirds of those children asked were able to do so, followed by the bus, explained by a half, and the phone at under a half. The tv control was explained by a third and the coin was found the most difficult, explained by less than a quarter of all children. Relationships between responses to why there is a number and age are shown within each country.

Separate countries

England

Results for English participants when asked why the numbers were there can be seen below.

Table 5.42 Overall frequencies of responses to "Why are there numbers there?" by English participants N=48

Item	N	Explained	Not explained
Coin	48	25	23
Phone	48	20	28
Bus	48	25	23
Card	48	40	8

English children found explaining the number on the birthday card the easiest, explained by over four fifths. The bus was found just as difficult as the coin, explained by around a half, followed by the telephone at under a half. A Cochran Q test and follow-up McNemar tests ($p< .05$) confirmed that the number on the card was explained significantly more successfully ($Q = 25.47$, $df = 1$, $p =.000$) than the other three items.

Relationship between explanations for the number and age

Table 5.43 Relationship between responses to "Why are there numbers on the coin?" and age among English participants

Age groups (age in months)	N	Explained	Not explained
2: 42-49	24	9	15
4: 55-60	24	16	8

Even on the coin, older children in England were almost twice as successful as the younger children at explaining why the numbers are there. Likelihood ratio tests show that success in explanation of a numeral on the coin improved with age among English children(X^2 = 4.151, df = 1 p =.042).

Table 5.44 Relationship between responses to "Why are there numbers on the phone?" and age among English participants N=48

Age groups (age in months)	N	Explained	Not explained
2: 42-49	24	6	18
4: 55-60	24	14	10

Results for why there are numbers on the telephone in table 5.44 show that success rates more than doubled with age. Likelihood ratio tests also show that success in explanation of a numeral on the phone improved with age among English children (X^2 = 5.609, df = 1 p =.018).

Table 5.45 Relationship between responses to "Why are there numbers on the bus?" and age among English participants N=48

Age groups (age in months)	N	Explained	Not explained
2: 42-49	24	9	15
4: 55-60	24	16	8

Table 5.45 shows the results for the bus for the two age-groups of English children which indicates that improvement with age almost doubles. Likelihood ratio tests confirmed that success in explanation of a numeral on the bus improved with age among English children (X^2 = 4.151, df = 1 p =.042).

Table 5.46 Relationship between responses to "Why are there numbers on the birthday card ?" and age among English participants N=48

Age groups (age in months)	N	Explained	Not explained
2: 42-49	24	17	7
4: 55-60	24	23	1

In table 5.46 can be seen overall results for why there is a number on the birthday card, showing that more children in the older group were successful. Likelihood ratio tests confirm that success in explanation of a numeral on the card improved with age among English children ($X^2 = 5.965$, df = 1 p =.015).

Japan

Japanese children were asked to give reasons for the presence of numerals on the three items they had observed throughout the study.

Table 5.47 Overall frequencies of responses to "Why is the number there?" by Japanese children N=71

Item	N	Explained	Not explained
Coin	71	33	38
Phone	71	24	47
TV control	71	23	48

The responses, shown in table 5.47, show that around a third of the participants were able to explain the tv control and phone successfully, whereas almost a half were able to explain the coin. A Cochran Q test confirmed that Japanese children found the coin easiest (Q = 6.741, df = 2, p = .034) and follow up McNemar tests showed that the coin was found significantly easier than the tv control (p =.000) and easier than than the phone at a level approaching significance (p =.093).

Relationship between explanations for the number and age

Responses were examined for any changes with age according to each item.

Table 5.48 Relationship between responses to "Why are there numbers on the coin?" and age among Japanese participants N=71

Age groups (age in months)	N	Explained	Not explained
2: 42-49	10	2	8
3: 50-54	34	14	20
4: 55-60	22	13	9
5: 61-62	5	4	1

Results for the coin, in table 5.48 above show that a fifth of the youngest children were able to say why there is a number on the coin, rising to around a half and then to around three-quarters of the oldest children, although this is a small group. Likelihood ratio tests show that success in explanation of a numeral on the coin improved with age among Japanese children at a level approaching significance (X^2= 7.226, df = 3 p =.065).

Table 5.49 Relationship between responses to "Why are there numbers on the phone ?" and age among Japanese participants N=71

Age groups (age in months)	N	Explained	Not explained
2: 42-49	10	0	10
3: 50-54	34	12	22
4: 55-60	22	8	14
5: 61-62	5	4	1

In table 5.49 it can be seen that results for why there are numbers on the phone also show an improvement with age, from zero among the youngest to a third and then to over three-quarters of older participants able to do so. Likelihood ratio tests confirm that success in explanation of numerals on the phone improved significantly with age among the Japanese children (X^2 = 12.846, df = 3 p =.005).

Table 5.50 Relationship between responses to "Why are there numbers on the TV control?" and age among Japanese participants N=71

Age groups (age in months)	N	Explained	Not explained
2: 42-49	10	1	9
3: 50-54	34	9	25
4: 55-60	22	10	12
5: 61-62	5	3	2

In table 5.50 above, the results for why there are numbers on the tv remote control indicate a definite improvement with age, rising from under a tenth to a third then to two-thirds of children in the oldest group. Likelihood ratio tests also show that success in explaining numerals on the tv control improved with age among Japanese children at a level which approached significance (X^2 = 6.586, df = 3 p =.086).

Sweden

In table 5.51 can be seen overall results for why there are numbers on the four items shown to Swedish participants.

Table 5.51 Overall relationship between responses to "Why are there numbers there?" among Swedish participants N=48

Item	Explained	Not explained
Coin	6	42
Phone	2	46
Bus	4	44
Card	29	19

The above results show that overall the number on the birthday card was the easiest to explain, with the bus, card and phone found far more difficult. A Cochran Q test and follow-up McNemar tests confirmed that the card was found significantly easier than the three other items (Q = 53.467, df = 3, p = .000).

Relationship with age

Table 5.52 Relationship between responses to "Why are there numbers on the coin ?" and age among Swedish participants N = 48

Age groups (age in months)	N	Explained	Not explained
1: 36-41	8	0	8
2: 42-49	17	1	16
3: 50-54	10	2	8
4: 55-60	13	3	10

Results for the coin overall are very low, seen in table 5.52; although an improvement can be seen the numbers involved are very small, and likelihood ratio tests shows no significant improvement with age on the coin (X^2 = 4.510, df = 3).

Table 5.53 Relationship between responses to "Why are there numbers on the phone ?" and age among Swedish participants N = 48

Age groups (age in months)	N	Explained	Not explained
1: 36-41	8	0	8
2: 42-49	17	1	16
3: 50-54	10	0	10
4: 55-60	13	1	12

Table 5.53 shows the very poor results among Swedish children when giving reasons for numbers on the telephone, with no improvement with age. Likelihood ratio tests also show no significant improvement with age on the phone (X^2 = 1.970, df = 3).

Table 5.54 shows results for why there are numbers on the bus.

Table 5.54 Relationship between responses to "Why are there numbers on the bus ?" and age among Swedish participants N = 48

Age groups (age in months)	N	Explained	Not explained
1: 36-41	8	0	8
2: 42-49	17	1	16
3: 50-54	10	1	9
4: 55-60	13	2	11

The table above shows that Swedish children of all ages found it difficult to explain numerals on the bus, with very low numbers, and likelihood ratio tests shows no significant improvement with age on the bus (X^2= 2.266, df = 3).

Table 5.55 Relationship between responses to "Why are there numbers on the card ?" and age among Swedish participants N = 48

Age groups (age in months)	N	Explained	Not explained
1: 36-41	8	6	2
2: 42-49	17	11	6
3: 50-54	10	4	6
4: 55-60	13	8	5

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= 2, p = .060). In the older age-group Swedish children were significantly less successful than those in England and Japan at explaining the numeral on the coin, (X^2 = 6.788 df = 2, p =.034).

Table 5.57 Frequencies of responses to "Why are there numbers on the phone?" according to country and age-group

Age groups (age in months)	N	Explained	Not explained
2: 42-49	51		
England	24	6	18
Japan	10	0	10
Sweden	17	1	16
4: 55-60	59		
England	24	14	10
Japan	22	8	14
Sweden	13	1	12

As table 5.57 shows, among younger children no Japanese and only one Swedish participant were able to give reasons for numbers on the telephone, whereas a quarter of English children were able to do so. Among older children the picture in Sweden hardly changed whereas in Japan over a third were successful and in England the figure rose to over a half.

Pearson Chi-Square tests show that in the younger age-group English children were more successful at explaining the numeral on the phone, at a level which approaches significance (X^2 = 5.050df = 2, p =.080). Among the older group, English children were also significantly more successful (X^2 = 9.193 df = 2, p =.010).

Results for the other two items shown to participants in England and Sweden, the bus and the birthday card, are shown below.

Table 5.58 Frequencies of responses to "Why are there numbers on the bus?" according to country and age-group

Age groups (age in months)	N	Explained	Not explained
2: 42-49	41		
England	24	9	15
Sweden	17	1	16
4: 55-60	37		
England	24	16	8
Sweden	13	2	11

Table 5.58 shows that among the younger children, participants in Sweden were far less likely to explain numbers on the bus, whereas in England over a third were able to. There was an improvement with age in both England and Sweden; figures in Sweden were very low but in England the success rate rose to almost two-thirds.

Pearson Chi-Square tests show that in the younger age-group English children were more successful than Swedish at explaining the numeral on the bus, (X^2 = 5.394 df = 1, p =.020). Among the older group English children were also significantly more successful (X^2 = 8.877 df = 1, p =.003).

Table 5.59 Frequencies of responses to "Why are there numbers on the birthday card?" according to country and age-group

Age groups (age in months)	N	Explained	Not explained
2: 42-49	41		
England	24	17	7
Sweden	17	11	6
4: 55-60	37		
England	24	23	1
Sweden	13	8	5

Showing greater success overall on the birthday card, table 5.59 indicates that almost three-quarters of English children were able to explain the number on card and almost two thirds of Swedish. Performance hardly changed with age in Sweden however, whereas in England the figure rose to over ninety per cent among older children.

Pearson Chi-Square tests confirm that in the younger age-group there was no significant difference between English and Swedish children when explaining the numeral on the card, (X^2 = .173 df = 1, p =.678 ns) and that among the older group English children were significantly more successful than the Swedish (X^2 = 7.300 df =1, p = .007).

5.4 Overview of results of the Magic Book

Whereas children's understanding of cardinal aspects of number were investigated in the tins game, milk note and fast food tasks, the Magic Book tasks extended the range of meanings investigated by also testing for ordinal and nominative aspects of number.

This task took into account both graphic response and spoken responses, as in the milk note and fast food tasks, but went further than most tests of number conducted with young children as it also asked for their views on the purpose and meaning of the numerals. This then allowed an examination of the extent to which children were able to recall and write that numerals were missing from the items, and the extent to which they understood their meaning. The items selected for the study were a familiar and meaningful part of the everyday lives of the children in each cultural setting.

Results for 'what's missing' for the two items shown to participants in all three countries show that overall, children were more likely to identify what was missing from the phone than the coin. There was only an improvement with age on the phone among English children.

On the two other items shown only to English and Swedish participants, the Swedish children were more able to say what was missing from the birthday card, but with no improvement with age on any item. Only the English children showed growing awareness with age that numbers were missing, and that was on the phone and birthday card.

Just as children in all three countries had found it hardest to identify that a numeral was missing from the coin, they also found it hardest to put in the numeral missing from the coin, with no improvement with age. Children in all three countries improved with age when asked to put in what was missing on at least one of the items.

Whereas children in all three countries had found the coin the most difficult on both previous tasks, when asked to explain their purpose the three countries differed. In the younger group of children aged 42-49 months the English children found the coin easier to explain than the Japanese or Swedish children who showed no significant difference. Among the older group aged 55-60 months the Swedish children found the coin significantly more difficult to explain than the English or Japanese.

Here a pattern can be identified, as children in England and Japan showed both an improvement with age in identifying the coin and explaining the coin, whereas older children in Sweden were correspondingly weaker on both questions. This suggests a close relationship between children's understanding of the meaning of a number and the ability to recall its presence in everyday settings.

Whereas English and Japanese children improved in their ability to explain the purpose of the numerals on all items with age, Swedish participants did not show any improvement with age on the four items shown to them. Similarly, although in the younger age-group there was no difference between England and Sweden in explaining the birthday card, in the older age-group the English children were better than the Swedish participants at explaining all four items. However it is interesting to note that although the English participants appeared to know or understand more about numerals, as expressed in their explanations, they were just as likely as other children to put them in.

In summary, the results on the Number Labels tests show that overall there were more differences between countries in their performance on these tasks involving cardinal, ordinal and nominal meanings than on the tins, milk and fast food tasks which involved cardinal number only.

The findings of the Number Labels task also provide data which could further explore the relationship between responses to the three questions to be examined. Although a causal link between a child's ability to recall and write a missing numeral and their understanding of its meaning could not be inferred on this basis of these findings, a follow-up study might use the data for a closer examination of these aspects of environmental number.

To facilitate an overview of the most significant findings a summary chart is provided below.

Table 5.60 Summary of results of the Magic Book

Participants	Aspect of performance	Significant findings	Changes with age
Overall	**"What's missing from the Coin and the Phone ?"** (2 common items)	Performance on the Phone was better than on the Coin.	
England	"What's missing from the Coin, Phone, Bus, Birthday Card?"	Performance on the Coin was the worst.	No improvement with age for Coin. Some improvement with age on the Phone and Card.
Japan	"What's missing from the Coin, Phone, TV Control ?"	Performance on the Phone was best but worst on the Coin.	No improvement with age.
Sweden	"What's missing from the Coin, Phone, Bus, Birthday Card ?"	Performance best on the Birthday Card.	No improvement with age on any item.
Comparing performance in England, Japan and Sweden	**"What's missing from the Coin and Phone?"** (2 common items)	Japanese children did least well on both the Coin and Phone.	
Comparing performance in England and Sweden	**"What's missing from the Bus and the Birthday Card ?"** (2 common items)	In the *younger age-groups only* Swedish did better than English children on both the Bus and Card. In *older groups* there was no difference.	

Participants	Aspect of performance	Significant findings	Changes with age
Overall	**"Put in what's missing from Coin, Phone."** (2 common items)	No significant difference between items.	
England	"Put in what's missing from the Coin, Phone, Bus, Birthday Card."	Performance on the Coin was the worst. Performance on the Phone. Performance on the Bus.	No improvement with age. Improvement with age. Improvement with age.
Japan	"Put in what's missing from the Coin, Phone, TV Control."	Performance on the Coin was the worst. Performance on the Phone.	No improvement with age. Improvement with age.
Sweden	"Put in what's missing from the Coin, Phone, Bus, Birthday Card."	Performance on the Coin was the worst. Performance on the Phone was best. Performance on the Bus and Birthday Card.	No improvement with age. Some improvement with age. No improvement with age.
Comparing performance in England, Japan and Sweden	**"Put in what's missing from the Coin and Phone."**	No difference between countries on any item in either age group.	
Comparing performance in England and Sweden	**"Put in what's missing from the Bus, and the Birthday Card."**	No difference between countries on any item in either age group.	

Overall	**"Why is there a number on the Coin and Phone?"**	No significant differences.	
England	"Why is there a number on the Coin, Phone, Bus and Birthday Card?"	Performance on the Card was best Performance on the Coin, Phone and Bus	Improvement with age Improvement with age
Japan	"Why is there a number on the Coin. Phone and TV Remote Control?"	Performance on the Coin was best Performance on the Phone and TV Remote Control	Some improvement with age Improvement with age
Sweden	"Why is there a number on the Coin, Phone, Bus and Birthday Card?"	Performance on the Card was best Performance on the Coin, Phone and Bus	No improvement with age No improvement with age
Comparing performance in England, Japan and Sweden	**"Why is there a number on the Coin and Phone?"**	Among the *younger age-groups*, best performance on the Coin was by English children Among the *older age-groups,* the worst performance on the Coin was by Swedish children Performance on the Phone was better by English children of all ages	
Comparing performance in England and Sweden	**"Why is there a number on the Bus and Birthday Card?"**	Performance on the Bus was better by English children of all ages Among the *younger age-groups*, there were no differences in performance on the Card. Among the *older groups*, the best performance on the Card was by English children	

Chapter 6

Results of tests involving nominal, cardinal and ordinal uses of number: the Party Invitation

In this test, participants were asked to respond orally to each entry on an Invitation card and then asked to 'write' it in the appropriate space. Their oral responses were then coded according to how numbers and calendar words were used within the context of each item on the invitation; oral responses were considered the easiest and most reliable way for such young children to communicate their understanding about these complex issues.

6.1 Coding of responses

The coding of 'Full' oral use was awarded when a child used the numbers and words in same way that an adult would conventionally use them. For example, when responding to the date if a child gave both the day and the month together, e.g. "the tenth of June", the code full was given. For the time, it was coded as full use when the child replied for example "at six o'clock" or "at half past five". For the address, full was coded when the number of the house was given plus the road, or the floor number and the name of the block of flats. For the telephone number full was awarded when at least four digits were given.

The coding of 'Partial' use was given when the child's response was in a shortened form which might indicate a degree of understanding. When asked the time, for example, if a child replies "… seven", it could be assumed that he is answering in the way that an experienced adult might say "at seven", however this reply was coded as partial as it is not certain that the child is communicating a time. Similarly for the date, a response was judged as partial if the child gave only one part of the date e.g. "June" or "tenth". In the address it was coded as partial if either only a number was given or only the street or district; for example if the child reply "number ten", "Wakefield Gardens" or "Kingsthorpe". Use of the telephone number was considered to be partial if three digits were given.

In the case of the Japanese language, the correct date involves the use of two ordinal numbers, one for the day and one for the number of the month. Therefore the Japanese children's use of two numbers in their answers was judged as full, and the use of one number as partial for example "the tenth" or "the sixth month". Similarly, because many Japanese children live in districts and apartment blocks which are numbered, the child's use of a conventional address was coded as full; the use of one name or number, such as in "fourth block" or "Sayama Mansion", was coded as partial.

In many cases the children appeared to understand the question and tried hard to communicate what they knew; for example for the address one Japanese child attempted to draw a map and another gave directions with a description of the place. However, for the purposes of this study these answers were coded as incorrect.

The data were analysed using two forms of scoring. One form gave credit for partial descriptions. The other only considered full descriptions as successful. The reasons for doing this are:

1) only in full descriptions is a number always required for each item;

2) adequate partial descriptions vary with country. For Japanese children even partial descriptions of a date had to feature a number (the number of the month and the number of the day of the month) whereas for English and Swedish children a partial date might just be the name of the month.

6.2 Overall results

Overall frequencies of responses to each item on the Invitation are shown in Table 6.1

Table 6.1 Overall frequencies of different forms of representation used on the Invitation N=167

Item:	Full	Partial	Incorrect
Date	41	60	66
Time	109	28	30
Address	50	47	70
Telephone Number	39	12	116

Overall, as the table shows, participants found time the easiest to respond to, answered fully by over two-thirds of children; when considering those who were partially accurate, it shows that over three quarters were able to provide an answer to 'what time'?

Less than a third of the participants fully answered the other three items. The address was answered fully by almost a third, and altogether over a half were able to answer the question, at least partially. Overall this was similar to the date because whereas a quarter of participants replied fully, well over a half were able to reply to some extent. The telephone number was least successful overall with less than a third giving an appropriate reply.

Cochran Q tests show that when giving full responses, time was answered best by participants overall (Q= 101.884, p=.000); follow-up McNemar tests confirmed that performance on the other three items did not differ significantly. When partial responses were also taken into account, time was again answered best (X^2 = 117.011, df = 3, p = .000); follow up Wilcoxon tests showed that overall participants found the time the easiest (p = .000) followed by the date, which was significantly more difficult than the telephone number which gave the poorest responses (p = .000).

Results for each country were examined separately in order to identify any differences in performance. The relationship between overall responses and age groups are shown within each country.

6.3 Results according to country

The results for each country are shown below in table 6.2

England

Table 6.2 Frequencies of responses to different items given by English participants N=48

Items:	Full	Partial	Incorrect
Date	16	16	16
Time	37	5	6
Address	23	12	13
Telephone Number	15	4	29

Overall responses for English children show that over three-quarters were able to answer the time. A half were able to give the full address, with a further quarter able to give a part of it. The date was next with a third successful and another third giving a partial reply. Nearly a third also gave a full response for the telephone number, but very few gave a partial response making the overall response rate for the telephone number the lowest.

Whether partial descriptions are taken into account or not, the English children showed substantial variation in the quality of descriptions according to item (Full only, Cochran $Q = 31.13$, $df = 3$, $p < .001$; Partial, Friedman $X^2 = 34.89$, $df = 3$, $p < .001$).

Follow-up McNemar tests for full responses only showed descriptions of time were better than the rest ($p< .01$) and that the address tended to be answered better than the telephone number at a level approaching significance ($p <.077$).

When partial descriptions were taken into account, Wilcoxon follow-up tests showed that time was found the easiest ($p=<.01$) and that both address and date descriptions were better than those for the phone number ($p < .05$).

Relationship between responses and age

Age trends were also examined according to the relationship between responses given for each item and age by the English participants are shown in Table 6.3

Table 6.3 Responses by the different age groups of English children to the Date N=48

Age groups	N=	Full	Partial	Incorrect
2: 42-49	24	6	6	12
4: 55-60	24	10	10	4

Among the younger English children a half were able to give some information about the date, and a quarter did so successfully. This improved with age as over three quarters of older children replied and almost a half gave a full response.

Spearman non-parametric tests on full responses show that for the date, age was not significant among the English children ($r_s = .163$, $p = .269$ ns). When partial responses to the date are taken into account however, performance was significantly related to age ($r_s = .296$, $p = .041$).

Table 6.4 Responses by the different age groups of English children to the Time N=48

Age groups	N=	Full	Partial	Incorrect
2: 42-49	24	14	4	6
4: 55-60	24	23	1	0

On the time, three quarters of younger children were able to respond and over a half gave a full reply. There was a marked improvement with age as almost all older children gave a full response.

Parametric tests show that in giving a full response for the time, age was significant among the English children ($r_s = .471$, $p = .001$). When partial responses to the time are taken into account there was also a significant relationship between age and performance ($r_s = .485$, $p = .000$).

Table 6.5 Responses by the different age groups of English children to the Address N=48

Age groups	N=	Full	Partial	Incorrect
2: 42-49	24	9	8	7
4: 55-60	24	14	4	6

On the address, two thirds of younger English children replied, over a third successfully. There appeared to be some improvement with age as although the number replying was similar, over a half of the older children did so fully.

However, tests show that performance on the address was not significantly related to age among the English children for either full responses (r_s = .175,) or partial responses (r_s = .154).

Table 6.6 Responses by the different age groups of English children to the Telephone Number N=48

Age groups	N=	Full	Partial	Incorrect
2: 42-49	24	6	0	18
4: 55-60	24	9	4	11

On the telephone a quarter of children of responded, all of whom gave a full reply. There was an improvement with age as over half the children responded appropriately and over a third gave a full reply.

Tests show that in giving a full response for the telephone number, English children's performance was not related to age (r_s = .121), and for partial responses there was a relationship to age approaching significance (r_s = .241, p = .099).

Japan

Table 6.7 shows the overall responses on the four items on the invitation.

Table 6.7 Overall frequency in the responses given by Japanese children for each item N= 71

Items	Full	Partial	Incorrect
Date	18	32	21
Time	55	0	16
Address	10	21	40
Tel Nr	18	3	50

Over three quarters of Japanese children gave a full reply to the time, but none partially. Next was the date, with three quarters responding appropriately and a quarter giving a full reply. A quarter gave a full reply to the telephone number and just three other children gave a partial reply.

The address was found more difficult, as although a half gave a relevant response, only a seventh of participants gave a full reply.

Whether or not partial descriptions are taken into account, the Japanese children's responses varied widely in the quality of descriptions according to item (Full only, Cochran Q = 76.03, df = 3, $p < .001$; Partial, Friedman X^2 = 57.399, df = 3, $p < .001$). Follow-up McNemar tests showed full descriptions were better for time than for the other three items ($p = .000$) and that responses for the address tended to be greater than for the telephone number at a level which approached significance ($p = .077$). When partial descriptions were taken into account, Wilcoxon tests showed that descriptions of the time were better than the rest and descriptions of the date were better than either the address or phone number (all p levels < .001).

Relationship between responses and age

Table 6.8 Responses by the different age groups of Japanese children to the Date

Age groups	N=	Full	Partial	Incorrect
2: 42-49	10	0	6	4
3: 50-54	34	8	14	12
4: 55-60	22	7	11	4
5: 61-62	5	3	1	1

On the date, although over half of the youngest Japanese children gave an appropriate response none did so fully, whereas this improved steadily with age as a quarter of group 3 answered fully, a third of group 4 and two thirds of group 5, with many more giving partial responses.

Parametric tests show that in giving both full and partial responses for the date, performance was significantly related to age among the Japanese children (Full, $r_s = .386$, $p = .001$; partial, $r_s = .356$, $p = .002$).

Table 6.9 Responses by the different age groups of Japanese children to the Time

Age groups	N=	Full	Partial	Incorrect
2: 42-49	10	5	0	5
3: 50-54	34	26	0	8
4: 55-60	22	20	0	2
5: 61-62	5	4	0	1

Results for the time are interesting as no Japanese children gave a partial response. All appropriate responses were full with a half and three quarters in the two younger groups and nearly ninety per cent among the two, smaller, older groups.

Spearman non-parametric tests show that for both full and partial responses for the time, age was not significant among the Japanese children (full, $r_s = .175$, $p = .143$ ns; partial, $r_s = .175$, $p = .143$ ns).

Table 6.10 Responses by the different age groups of Japanese children to the Address N=71

Age groups	N=	Full	Partial	Incorrect
2: 42-49	10	2	2	6
3: 50-54	34	3	12	19
4: 55-60	22	4	5	13
5: 61-62	5	1	2	2

Results for the address, in table 6.10 are quite different as in most age-groups more children gave a partial reply than a full one. But for the full replies there is hardly any improvement with age as only a ninth of children replied fully in the two younger groups and a fifth in the two smaller older groups.

Spearman non-parametric tests show that in giving full and partial responses for the address, performance was not significantly related to age among the Japanese children (full, $r_s = .061$; $r_s = .0322$).

Table 6.11 Responses by the different age groups of Japanese children to the Telephone Number N=71

Age groups	N=	Full	Partial	Incorrect
2: 42-49	10	2	0	8
3: 50-54	34	8	2	24
4: 55-60	22	5	1	16
5: 61-62	5	3	0	2

As table 6.11 shows, responses overall for the telephone number are low. Only a fifth of the youngest group gave a relevant reply and a third of the second group. Among the two older groups under a third gave a full reply, suggesting hardly any improvement with age.

Spearman non-parametric tests show that for both full and partial responses for the telephone number, there was no relationship between age and performance among Japanese children (full, $r_s = .122$; partial, $r_s = .111$).

Sweden

Table 6.12 Overall frequency in the responses given by Swedish children for each item N= 48

Items	Full	Partial	Incorrect
Date	7	12	29
Time	17	23	8
Address	17	14	17
Tel Nr	6	5	37

Overall results shown in table 6.12 indicate that Swedish children found time the easiest to answer, as over a third gave a full response and a half a partial response. A third also gave a full response for the address and just over a quarter gave a partial response. The date was a little harder as less than a seventh of participants replied fully while a quarter gave a partial reply. The telephone number was the most difficult as less than a quarter overall gave a relevant response.

Whether partial descriptions are taken into account or not, the quality of descriptions of the Swedish children varied with item (Full only, Cochran Q = 17,26, df = 3, p = .001; Partial, Friedman X^2 = 46.14, df = 3, p < .001). Follow-up McNemar tests for full responses only showed responses for the time and address were significantly better than the telephone number and date (p=<.05). When partial descriptions were taken into account, Wilcoxon tests showed that time was better than address (p =.014) and that the address was better than the date and telephone number (p <.01).

Relationship between responses and age

Table 6.13 Responses by the different age groups of Swedish children to the Date N=48

Age groups	N=	Full	Partial	Incorrect
1: 36-41 months	8	1	2	5
2: 42-49	17	0	3	14
3: 50-54	10	3	3	4
4: 55-60	13	3	4	6

As table 6.13 shows, both full and partial responses for the date by Swedish children were generally low, with slightly more children giving partial than full replies. In the two youngest groups less than a quarter answered appropriately, and just over a half of children in the two older age-groups were able to give a full or partial reply for the date.

Parametric tests show that in giving full and partial responses for the date, performance was not significantly related to age among the Swedish children (full, $r_s = .186$; partial, $r_s = .187$).

Table 6.14 Responses by the different age groups of Swedish children to the Time N=48

Age groups	N=	Full	Partial	Incorrect
1: 36-41 months	8	1	5	2
2: 42-49	17	2	11	4
3: 50-54	10	5	4	1
4: 55-60	13	9	3	1

Responses to the time were a little better than for the date. Among the two younger groups about two-thirds gave a partial response, while only an eighth of those in the two younger groups answered fully. There was an improvement with age however as in the third group a half gave a full reply and in the oldest around seventy per cent did so, with others giving a partial response.

In giving a full response for the time, Spearman non-parametric tests show that among the Swedish children, performance was significantly related to age ($r_s = .443$, $p = .002$). Partial responses too were also related to age ($r_s = .380$, $p = .008$).

Table 6.15 Responses by the different age groups of Swedish children to the Address N=48

Age groups	N=	Full	Partial	Incorrect
1: 36-41 months	8	2	3	3
2: 42-49	17	3	5	9
3: 50-54	10	4	3	3
4: 55-60	13	8	3	2

On the address there was also an improvement with age, as a fifth of younger children gave a full reply and a third partially. In the third group just under a half answered fully and a third partially, whereas in the oldest age-group about another quarter of children answered partially but nearly two-thirds answered fully.

Non-parametric tests show that in giving a full response for the address, among the Swedish children was significantly related to age (r_s = .308, p = .033). For partial responses the relationship with age was a level approaching significance (r_s = .280, p = .054).

Table 6.16 Responses by the different age groups of Swedish children to the Telephone Number N=48

Age groups	N=	Full	Partial	Incorrect
1: 36-41 months	8	1	2	5
2: 42-49	17	0	1	16
3: 50-54	10	2	0	8
4: 55-60	13	3	2	8

Swedish participants giving either a partial or full reply to the telephone number were overall very low, with only a sixth among the two younger groups and less than a third in the older groups.

For both full and partial responses for the telephone number, parametric tests show that among the Swedish children performance was not related to age (full, r_s = .224; partial r_s = .147).

6.4 Comparison of countries

A comparison of the relative performance of the three nationalities was made using the younger and older age-groups 2 and 4. Frequencies of responses to each item on the invitation according to the two age groups can be seen in Table 6.17

Table 6.17 Comparison of responses to the Date by the different age groups

Age groups	N=	Full	Partial	Incorrect
2: 42-49 months	51			
England	24	6	6	12
Japan	10	0	6	4
Sweden	17	0	3	14
4: 55-60	59			
England	24	10	10	4
Japan	22	7	11	4
Sweden	13	3	4	6

Among the younger groups English participants were far more successful in responding to the date, mainly as none of the Japanese or Swedish children answered fully. A further quarter of English children replied partially, over a half of Japanese and less than a fifth of Swedish children.

English children were also more successful in the older age-group as almost a half answered the date fully, compared to a third of Japanese and a quarter of Swedish children. A half of the children in England and Japan gave a partial response to the date and just under a third in Sweden.

Kruskal-Wallis non parametric tests show that in the younger age-group there was a significant difference between countries for the date (X^2 = 6.937, df= 2, p=.041), with younger Swedish children faring worst. However there was no difference between countries among the older children when replying to the date (X^2 = 3.286, df = 2, p = .193).

Table 6.18 Comparison of responses to the Time by the different age groups

Age groups	N=	Full	Partial	Incorrect
2: 42-49 months	51			
England	24	14	4	6
Japan	10	5	0	5
Sweden	17	2	11	4
4: 55-60	59			
England	24	23	1	0
Japan	22	20	0	2
Sweden	13	9	3	1

When asked to give the time, the younger English participants were the most successful in giving a full reply at almost sixty percent, and a sixth answered partially. A half of younger Japanese children answered fully but none partially. However among Swedish children only an eighth gave a full reply although over sixty percent gave a partial response.

In the older age-group all English children replied appropriately for the time, almost all fully. Older Japanese children were almost as successful and seventy per cent of Swedish children gave a full reply and another quarter a partial response.

Kruskal-Wallis non-parametric tests show that in the younger age-group there was no significant difference between countries for the time (X^2 = 3.554, df= 2, p= .169 ns). In the older age-groups of the three countries, there was a difference between countries at a level approaching significance (X^2 = 5.558, df = 2, p = .062), mainly due to the poorer responses of the older Swedish children.

Table 6.19 Comparison of responses to the Address by the different age groups

Age groups	N=	Full	Partial	Incorrect
2: 42-49 months	51			
England	24	9	8	7
Japan	10	2	2	6
Sweden	17	3	5	9
4: 55-60	59			
England	24	14	4	6
Japan	22	4	5	13
Sweden	13	8	3	2

On the address younger English children fared best with over a third giving a full reply and another third a partial reply. Only a fifth of younger Japanese children gave either full or partial responses; in Sweden nearly a sixth replied fully and nearly a third partially to the address.

Among the older age-group, nearly two thirds of the English and Swedish children gave a full response, compared to less than a fifth of Japanese children. Partial responses were about the same across countries at around a fifth.

Kruskal-Wallis non parametric tests show that in the younger age-group there was no difference between countries for the address (X^2 = 3.864, df= 2, p=.145 ns). Between older children, there was a significant difference between responses to the address in the three countries (X^2 = 10.843, df = 2, p = .004), with Japanese children the least successful.

Table 6.20 Overall Responses by the different age groups to the Telephone Number

Age groups	N=	Full	Partial	Incorrect
2: 42-49 months	51			
England	24	6	0	18
Japan	10	2	0	8
Sweden	17	0	1	16
4: 55-60	59			
England	24	9	4	11
Japan	22	5	1	16
Sweden	13	3	2	8

When asked for the telephone number, results for the picture are similar to the address among younger participants. The younger English children were more successful as a quarter replied fully and none partially, whereas only a fifth of Japanese children gave a full reply and none a partial response. None of the Swedish replied fully and only partially.

Among the older group the English children were more successful as just under a third gave a full reply and another sixth a partial reply. Nearly a third of older Japanese children gave full or partial replies for the telephone number, as well as almost a third of Swedish children.

Kruskal-Wallis non parametric tests show that in neither the younger nor older age-groups were there any differences between countries for the phone number (younger, $X^2 = 2.882$, df= 2, p= .237; older, $X^2 = 2.896$, df = 2, p = .235).

6.5 Overview of results of the Invitation task

The task of completing an invitation was devised as it aims to draw upon the party as a familiar and salient event in young children's lives. The task explored children's knowledge of other everyday uses of number: date, time, address and telephone number in a context that again involves communication with another person.

It is important to note that the design of the party invitation could not have been identical to invitation cards previously seen by each child in the course of everyday life, yet the design was acceptable to participants in all three countries as they all responded to the invitation task and questions.

Overall, participants answered the time question better than the other items, both in full and partially, with no major differences between countries although the older Swedish children were a little less successful. Results show that time was the most successful response, despite the complexity of the language of 'o clock' and 'half past'; the same can be said about the date, yet this was found second easiest. Greater success on the time question may reflect socio-cultural influences, as time is a topic that is mentioned frequently throughout the day in the home, particularly where family members need to consider time-keeping; as all children participating in this study regularly attended a nursery school this would apply to their families. Furthermore, in nursery classes staff keep to a timetable of activities which may provide further experience of telling the time.

When partial responses were also taken into account, the date was found the next easiest. Reasons for this seem less obvious. Although older children in English primary schools are required to write the date in their books every day, it cannot be guaranteed that younger children in nursery settings would regularly be told the date. The date is perhaps most likely to be stated, in the home and nursery, in connection with birthday dates and the dates of a child's birthday party, as in this study.

In the three nursery schools in Japan in which this study was conducted, the birth date of each child was displayed on separate posters high on the walls of each classroom. However, the results show that whereas the younger Swedish children fared worst on the date, among the older children there was no significant difference between England, Japan and Sweden.For the address there was no difference between countries in the younger group, but in the older age-group the Japanese children were the least successful.

What is particularly striking about these results is that the telephone number was found to be the most difficult by participants in all three countries, with only Japanese children showing an improvement with age. After all, repetition of just three or four digits could be considered to be *less difficult* than recalling the time, date or address. There may be several reasons for this relative difficulty.

Firstly, such young children would rarely be asked for their telephone number as one would normally ask the adult or an older sibling. Secondly it is unusual to state the phone number when answering a telephone call, although this was common practice in England in the past, so children now rarely hear their phone number spoken. Telephone numbers seem unlikely to be the topic of conversations in the home which involve young children compared to say birthdays and parties, so perhaps they are outside the experience of most young children. Even if told a phone number, perhaps such young children would find it difficult to memorise a set of numbers which are not in the natural number sequence. Although young children may be encouraged to memorise rhymes and songs, they are rarely required to memorise a string of items that hold no meaning for them.

The fact that the results for the phone number are similar across countries may also suggest that everyday practices are the same in the three cultures. If the reasons given for the differences in performance between items are valid, this would seem to suggest that young children are learning a great deal about some aspects of number as a result of social or cultural exposure. If so, this would reflect different rates and types of exposure. Indeed, the high success rates for the time question indicate that a grasp of some adult uses of number in everyday life can be achieved at this young age.

6.6 Summary

The difficulties that these young children experienced when communicating cardinality to another person could be attributed to egocentrism, to a universal stage in cognitive development or to the result of socio-cultural experiences which exclude a demonstration of the relevance and power of numerals to communicate cardinal aspects of number. The similarities in results of tasks involving everyday uses of number could be attributed to either universal rates of development or to socio-cultural practice.

Firstly it is important to note that the design of the party invitation could not have been identical to invitation cards previously seen by each child in the course of everyday life, yet the design was acceptable to participants in all three countries as they all responded to the invitation task and questions. Secondly, results show that time was the most successful response, despite the complexity of the language of 'o clock' and 'half past'; the same can be said about the date, yet this was found second easiest. In contrast, repetition of two or three digits of the phone number involved simple number names, yet this was found the most difficult.

The similarities in results of tasks involving everyday uses of number could be attributed to either universal rates of development or to socio-cultural practice. It is important to note that results on the tasks which involved *cardinal* aspects of number became progressively lower (i.e tins game, milk note and fast food with the degree to which the child was required to communicate cardinality to another person. *The ability to count, write a numeral or give a cardinal number in response to a direct 'how many?', are not in question.* There may be a developmental delay in these young children's knowledge of how and when number is used in society for communicating cardinality. In contrast, in general their use of number for other purposes as in the Invitation and Number Labels tasks was relatively further developed.

In order to facilitate viewing of the results of the Invitation task a summary chart has been provided.

Table 6.21 Summary of results of the Party Invitation

Participants	Aspect of performance	Significant findings	Changes with age
Overall	**"What is the Date, Time, Address and Phone Number of the party?"**	Performance on the Time was best, followed by Date. The Phone Number was worst.	
England	"What is the Date, Time, Address and Phone Number of the party?"	Performance on the Time was best Followed by Address Followed by Date.	Improvement with age for Full and Partial No improvement with age No improvement with age for Full, but improvement when Partial taken into account
		The Phone Number was worst.	Some improvement for Partial only
Japan	"What is the Date, Time, Address and Phone Number of the party?"	Performance on the Time was best for Full. Followed by Date for Full and Partial. Followed by Address and Phone Number.	No improvement with age Improvement with age Phone Number, improvement with age Address, no improvement with age
Sweden	"What is the Date, Time, Address and Phone Number of the party?"	Time and Address better than Phone Number. When Partial taken into account, Time and Date were better than Phone Number.	Date, no improvement with age Time and Address improvement with age Phone number, no improvement with age
Comparing performance in England, Japan and Sweden	**"What is the Date.?"**	Among the *younger groups*, performance of the Swedish children was the worst. No difference among the *older age-groups*.	
	"What is the Time .?"	Among *younger age-groups* there was no difference. Among *older age-groups* Swedish a little worse.	

	What is the Address?"	Among the *younger groups* there was no difference. Among *older groups*, performance of Japanese children was worst.	
	What is the Phone Number?"	No difference between countries.	

Chapter 7
Discussion and Conclusion

7.1 Discussion

In this discussion of results it may be helpful to bear in mind that the design of this study aimed to examine aspects of the 'protracted' period of development of young children's knowledge and reasoning about number (Dowker & Cowan, 1998) and the 'multi-faceted' nature of their conceptual development. To this end the study was designed to access certain aspects of children's understandings about number, including the personal meanings that they hold, in order to begin exploring how young children see the world with regard to number.

The study therefore endeavoured to use broad and comprehensive tasks and materials (Cowan, 1991) and genuinely experimental designs with more detailed task analysis (Rittle-Johnson & Siegler, 1998). The study also investigated children's understanding of ordinal and nominal as well as cardinal uses of number, and was designed to take into account both the oral as well as graphical forms of representation used by very young children.

Although the study and its findings cannot provide causal evidence and or evidence of relations between concepts and procedures, it is hoped that the findings will contribute towards such investigation in the future. The following discussion endeavours to take a multi-faceted view to examining the representation, understandings and meanings of very young participants obtained by using comprehensive tasks and 'everyday' materials in different cultural settings.

7.1.1 The concept of cardinal number

Overall, the findings of the tins game in England, Japan and Sweden were similar to each other and similar to those obtained by Hughes (1986). The results showed increased use of iconic and symbolic forms of representation, but that the spontaneous use of numerals was still relatively unusual at ages 3 and 4.

It is interesting to note that labels were left blank solely on the tin with no blocks. The reasons for this are difficult to fathom. Firstly, it could signify a refusal to commit a graphical response to paper; this could be where the child does not know how to respond when there are no blocks, does not know whether any response is needed, or does not know whether there is any symbol or way of showing 'nothing', 'empty', 'nought' etc. In other words the child can offer no response or solution to the problem of representing a setting where there are no objects.

Secondly, a blank label could signify a child's pictographic or iconic attempt at depicting a tin visually, showing that no blocks can be seen inside. Thirdly, it could be that the child is fully aware that there are "no" blocks, that this needs to be represented by a symbol, but is conscious that she does not know what the symbol is. (It is unlikely that the child recalls the zero symbol but is unable to write it as it a simple symbol for a child to form.) In the latter case the child would not be prepared to invent her own symbol as she would be aware that it would not be conventionally accepted; the child's only option then is to leave the label unwritten. Whether the latter two responses could be deemed as more 'sophisticated' than a refusal based on not knowing is a moot point.

When the numbers of blank responses for zero are compared to the average numbers of symbolic responses the pattern varies. In England 14 labels were left blank for zero and 8 were symbolic, with an average of 15 symbolic responses on tins 1-3. In Japan the picture was the reverse and less marked: 27 labels were left blank for zero and 33 were symbolic - against an average of 36 for symbolic responses on tins 1-3 and with a greater number of participants. In Sweden the pattern was the reverse of England's: only 8 children left the label blank for zero and 12 used symbolic, alongside an average of 10 using symbolic on tins 1-3.

Although overall the numbers of participants using blank and symbolic forms of representation for zero varied between countries, it is interesting to observe that the numbers leaving the label blank on the empty tin were close to those using symbolic on tins 1-3 in every country, and that the accuracy rates for reading back the blank label were over 90% overall where tested, i.e. in England and Sweden.

It is therefore not easy to gauge the levels of maturity or sophistication of those participants leaving the label blank on the tin with no blocks, however it is interesting to note that more older children than younger children chose to leave the label on the empty tin blank. In age groups 1 and 2 25% of participants left the label blank while in age groups 3 and 4 this rose to 32%. When one recalls that children generally moved to using more mature forms of representation with age (e.g. symbolic) it may be considered that leaving the label blank may in fact be interpreted as a pictographic or iconic form of representation, or that the child is aware that she does not know the symbol for "no" blocks. Clearly, this hypothesis would require further investigation.

The results for writing a note to a Milkman or Shopkeeper show that the use of numerals was even lower. This is despite the fact that both tasks had been based on providing a clear purpose for children, as opposed to a more abstract task devoid of meaning for a young child. Results were no better for the English children, all of whom were familiar with the cultural practice of communicating about amounts of milk required to a tradesperson, than for children in Japan or Sweden where there is no such custom. Therefore the contexts of both a milkman and a shopkeeper receiving a note was even less effective in all three countries in eliciting children's use of numerals than the Tins Game.

In the Fast Food task, which was open-ended and required participants to respond solely orally about the amounts required, the children made even less use of cardinal number to represent the items. Indeed, of those who gave answers that were *generally* relevant to ordering food, 64 out of 83 did not give the cardinal number of items required.

However recent findings on the social aspects involved in children's number development need to be located within the broader research context. The low scores in the three applied number tasks can be considered against the findings of Frye et al (1989) who gives the average age of developing the concept of cardinal number as around one year after learning to count objects aloud, and to that of Wynn (1990, 1992,) who found it to be around 3 years 7 months, and that of Fluck & Henderson (1996) who estimated it to be at around 4 years 1 month.

On the one hand we may note we can see from the results of this study that the use of cardinal number actually falls with the extent to which the three tasks increasingly require more *social forms* of communication. This may be attributed to a universal stage in development as in the Piagetian model and to Allardice's suggestion (1977) that young children fail to communicate number due to their ego-centricity, because they do not see the other person's point of view about requiring information about quantity.

The findings may also be seen to add to the Piagetian debate concerning links between the failure to link counting and cardinality and innate developmental structures. On the other hand the relative rarity of the (applied) use of cardinal number in these tests appears to be consistent with the findings of Cowan (1987) that young children who know how to count made judgments inconsistent with their counting skill.

Wynn (1990) and Fluck (1995) found that children were able to distinguish between the questions 'count' and 'how many', however it is important to note that in all of the cardinal number tasks in this study participants were not placed in the position of providing one-word responses. They were required to solve the problems of graphical representation, in the Tins Game and Milk Note tasks, and of communicating cardinality orally in the Fast Food task without supporting their communication by physically pointing to the items of food.

For example, although the three and four year-olds in the study might be able to provide oral and written answers to 'how many?' *if directly asked to state or write the numbers*, they may be seen as not having utilisational competence (Gelman et al, 1986) in these applied, problem-solving situations. However, this may be attributed to the development of skills before principles (Briars & Siegler, 1984; Siegler, 1991a), to the delayed use of cardinal number (Fluck & Henderson, 1996), or to the 'inaccessibility' of young children's counting skills (Dowker & Cowan, 1998).

Dowker's & Cowan's (1998) conclusion is that children's success in their non-verbal tasks becomes either inaccessible for some reason or not readily translatable into number words. However due to the variation in responses according to the degree of social communication required we may also consider the extent to which children's understanding and application of number may be related to the (social) meanings that they have acquired, i.e. to other external, as well as internal, factors.

The social nature of the three cardinal number tasks and the associated level of responses may focus research attention on the socio-cultural aspects of young children's number development (Cowan, 1987), and the 'social mechanisms' of cognitive change (Light and Perret-Clermont, 1989). This includes the support provided by the culture and the goals for number use shared by adults in and beyond the home (Sophian, 1998).

One further consideration is whether children understand more about cardinal number than they are able to communicate. This possibility may focus research attention on the nature of young children's communication skills and whether limitations in theory of mind (Frith, 1989) prevent young children from making deductions and inferences as to what the other person does not know or needs to know, as in the case of the milkman, shopkeeper and fast-food assistant.

7.1.2 Socio-cultural support for cardinal number

In line with the socio-cultural view we can also consider possible reasons for Fluck & Henderson's findings (1996) that most three and four year-olds had to be prompted towards last number word repetition following a count of objects. It is interesting to note that this is consistent with the finding that caregivers rarely ask their young children 'how many' and only occasionally repeat the last or cardinal number. As the meaning and use of number words are so closely interrelated in the first few years (Solomon, 1989) it also appears to substantiate Durkin's view (1993) that it is caregivers' practice of pointing and uttering a word that leads to such problems (Solomon, 1989) as children try to discover what adults mean. The practice also relates to Hughes' (1986) finding that children do not make use of cardinal number because they allocate a cardinal number with one-to-one correspondence to each discrete item in the set, usually orally, rather than to the set as a whole. We are also reminded of Fluck's (1995) suggestion that children appear to attribute the last count word to task completion rather than to cardinality.

The finding that, of the 167 children tested 81 did not name the fast food items regardless of whether or not they gave the number, is an interesting one. On the one hand it may reflect Durkin's finding that caregivers only 'occasionally' use a noun to indicate the items referred to, alongside a cardinal number (for example "three cats") and only when the child is correct in counting (Durkin, 1993). The naming of items of food would seem to be a simpler task than counting, which again may suggest that young children do not have directly modelled to them how to use nouns with number words in their speech to convey quantity. As we have seen this only happens with those children who already use numbers.

Similarly, the low scores obtained on the three cardinal number tasks may be due to caregivers' support being 'incidental' rather than 'intentionally pedagogic' (Fluck 1995) and of an 'implicit' rather than 'explicitly instructional' teaching style (Durkin, 1993). This may be due to their assumption that their children already understand the purposes of counting and cardinality, as found by Fluck (1995). In the largely non-cardinal tasks of the Magic Book and Invitation, which prompted higher scores, it may be that the purposes of similar tasks had previously been made more explicit by adults in the home and school.

The results obtained in the Magic Book and Invitation tasks reflect the findings of socio-cultural studies that frequency of exposure and extensive experience of number use in the home leads to enhanced development, and the results of the Tins Game, Milk Note and Fast Food tasks that lower levels of involvement and complexity result in lower levels of understanding and competence (Saxe et al, 1987). Sinclair (1991) and Frydman & Bryant (1998) similarly conclude that it is the nature of the child's everyday experience of written numerals that results in lack of salience of cardinality. This view is in line with Durkin's claim (1993) that children rarely undertake counting before starting school because they need or want to find out how many objects there are, but only do so *when asked or instructed by an adult.*

In particular, if children believe that cardinal number words are only given in response to direct questions such as 'count' or 'how many', but do not apply to social situations which demand the need to represent quantity, this would account for the low levels of success in the tins game, milk note and fast food tasks.

Comparing the cardinal and non-cardinal results seem to suggest that, as far as cardinal number is concerned, innate factors are unlikely to be the only source of young children's limited use of cardinality, as young children have to discover what adults mean concerning cardinal number before they are able to make the meaning their own. In the counting procedure in particular, concepts, meanings and number words are inextricably linked, as Vergnaud (1982) suggests, and embedded in the context. This also implies a difficulty with adult mathematical language, as discussed by Durkin (1993), as young children cannot always know when familiar number words refer to a familiar or unfamiliar process. For example, when asked to 'count' the young child cannot be sure if she is required to recite a string of numbers in the correct order, as when rehearsing the natural number sequence, whether to say the numbers aloud or silently while pointing to objects, whether to repeat or give one number to denote cardinality or to signify that the task has been completed.

If the meanings are implicit in the task and the adult does not make these explicitly accessible to the child, the child will be less likely to grasp the meaning. The low scores obtained on the cardinal number tasks in this study therefore raise questions about educational practice in the nursery classes attended by participants in the three countries in this study, and perhaps beyond.

As Durkin (1993) and Fluck (1995) have pointed out, the actions and interactions of the adults are particularly significant in developing children's understanding; for example, Hughes points out that in the Tins Game (1986) the adult draws children's attention not to the blocks themselves, but to the different numbers of blocks in the tins, suggesting that performance in his study improved due to adult help. This view may make a case for examining the social mechanisms of cognitive change in young children, raised by Light and Perret-Clermont (1989), which includes how pre-school children acquire meaning about number.

In the three tests in this study children's performance on the tins game was a little better than on the milk note and fast food tasks, possibly due to adult interaction as on the first trial with unlabelled tins they shared with the child something of the meaning of the task, which may well have been new and unfamiliar, and the reason for placing labels on the tins. This was followed by the second trial when each tin was labelled in turn. The second trial provided another four opportunities to represent the numbers of blocks, whereas a second trial was not provided in either the milk note or fast food tasks. Furthermore, whereas the fast food order offered three opportunities to communicate numerosity, the milk note provided only one.

7.1.3 Language and meaning

Despite the extensive work stemming from Vygotsky and Bruner on the interrelated nature of language and thought and the generative role of language, relatively little attention has been paid specifically to the role of language in cognition with specific regard to early number knowledge - and with which to discuss these findings. However we do know that much appears to depend on the child's understanding of the new symbolic language and on whether it becomes part of his own linguistic and conceptual framework.

However, mathematical language (Hersocovics & Bergeron, 1984; Cockcroft, 1982) constitutes far more than (mathematical) terminology; spoken language is supported by non-verbal and speech messages, with an estimated 93% of spoken communication effected through non-verbal channels (Sage, 2000) including context, inference, intonation and body language. We still know little about how children learn about number through aural means, how they integrate the language of number into their everyday speech, understand plural nouns and number words etc. Much attention has been paid to children's success and failure in operations involving written symbols, but little attention paid to the linguistic-cognitive underpinnings of early number when conceptual frameworks are being formed, or to the generative role of communication and spoken language in cognitive development.

The literature shows that the relationship between successful task completion, the child's ability to understand the task and the concepts involved appears to be complex, and that the intuitive understanding and mathematical processes children use appear to be closely related to their grasp of the mathematical language (Hersocovics & Bergeron 1984). The findings of the three cardinal number tasks in this study appear to endorse the work of Light & Perret-Clermont (1989) who stressed the importance of the pragmatic, inter-subjective agreements-in-meaning and interpersonal discursive clues in young children's mathematical development.

Donaldson's (1978) suggestion that the child does not interpret words in isolation but 'interprets situations' (p.88) provides a framework for such discussions about the meanings children hold for tasks such as these, i.e. tasks which require *communication* either to themselves or to others about both the numbers of objects and what they are. Puttnam's view (1989) that the referents for cardinal number word are socially fixed offers an explanation of why caregivers' infrequent reference to either cardinal number or to the objects to which they refer may have resulted in the difficulties encountered by the children in this study.

7.1.4 The acquisition of meaning

The findings of the cardinal number tasks appear to prompt us to examine whether the meanings that many young children develop, in relation to the uses and concept of number within the socio-cultural contexts of the home, everyday life and the nursery school, equip them to deal with tasks which involve communicating cardinality, to themselves or others. This is especially relevant to situations with a genuine human purpose that makes meaningful sense (Donaldson, 1978).

The findings may parallel the 'gap' found by Munn (1994) between what adults and young children believe about number. It may be the case that the children involved in this study share similar experiences to those involved in Munn's study (1994), most of whom did not share the purposes of adults but saw counting as reciting numbers rather than for discovering quantity, and as 'purposeless'.

Because the child 'does not interpret words in isolation, but interprets *situations*' (Donaldson, 1978) the design of these tasks was given much consideration. It was initially thought that the tins game might be seen by children as similar to a nursery-school task, as not only did it involve cardinal number but it also involved the counting of the kind of blocks typically seen in educational settings. For this reason, the milk note and fast food tasks were created as more domestic, everyday or 'environmental' settings in which to further test children's understanding and use of cardinal number. However, it may be that the participants of this and Munn's (1994) study did not recognise the 'everyday' purposes of the tasks, as a result of limited prior experience in the home and nursery, did not feel an urge to communicate, and had difficulty using the language and symbols of cardinal number.

Nunes (1993, p.74) points out that mathematical knowledge appears to bear the marks of the 'social situation' in which it was originally acquired and through the types of symbolic representation that was used. If this is the case then we are provided with a clear reflection of the type of mathematical experiences provided for the young child - which casts further doubt on the nature of activities, adult interaction and support in the area of cardinal number in the home and pre-school settings.

7.1.5 Learning to use numbers and number symbols

As these findings show, children need to know 'how and when to use numbers' (Solomon, 1989). Hiebert (1988) refers to the first step as involving children connecting individual symbols to reference of quantity, using mathematical symbols and numerals, and of children thereby crossing bridges between the two and creating their own specific meanings. Similarly, Hughes (1986) discusses the need for young children to form links and to have the ability to 'translate' between the formal and informal.

Although the recording of cardinal number features highly in pre-school or foundation stage curricula, these findings show that young children often find the purpose and meanings of cardinal number to be difficult and slow to grasp, with only 'gradual comprehension of the function of numerals and a widespread lack of understanding' (Durkin, 1993), at least as far as cardinal number is concerned. We might then consider the very nature of the curricula and associated number activities.

Carpenter's (1986) reference to the 'rich network of relationships which saturate the symbol system', and Sinclair's & Sinclair's (1986) reference to the conceptual content of symbols, the range and richness of associated meanings, emphasize the influence that they have on the nature of the conceptualisations that young children form. However, these findings suggest that extracting meaning about cardinality from the situations they encounter in the home and nursery have proved difficult for the participants of the study, and indicate that far greater support is needed, notably in cardinal number.

How is it then that children come to acquire the range of meanings that they have about when and how to use number systems, and how to record cardinality? The research background regarding this question is somewhat impoverished, as how children learn about mathematical symbols has been relatively little studied. Indeed in 1987 Hurford stressed the link between language and symbols, commenting that linguists had paid very little attention to symbols, and that symbols had not featured in any major intellectual debate.

However it would appear that the work of linguists on how young children come to learn about words, meaning and reading processes has much to contribute in this area. For example, Yetta Goodman emphasises that a child's language develops in a social setting 'because of the human need to communicate and interact' in response to creative participation (Goodman 1980, p.3).

Kenneth Goodman's work on language and reading describes the young child's inner drive to communicate ideas, how this drive underpins the desire for reading, and the meaning that these hold for the child. In contrast, when Wray (1989, p.66) discusses the existence of a 'literate environment' he emphasises that the concept of an 'environment' is fairly meaningless without people who actively demonstrate when and where print is used, how it is used and what it is. In the context of reading and writing both Yetta Goodman (1980) and Wells (1981) endorse this view and describe how children are often involved in adult activities and conversations in the home, mostly on a one-to-one basis.

Kenneth Goodman emphasises the close relationship between language and early literacy and suggests that language is easy to learn when it meets a 'functional need the child feels' (Goodman 1986, p.18). However this contrasts with evidence concerning young children's learning about number and. taking Goodman's view into account, suggests that young children find (mathematical) language hard to grasp and understand when it does not meet such a 'functional need'. This hypothesis appears to be echoed in Munn's findings that participants see counting as 'purposeless', and by the low results on the cardinal number tasks in this study which *had a clear purpose and required communication to another person.* The hypothesis also endorses Durkin's claim (1993) that children rarely undertake counting before starting school because they need to find out how many objects there are, but only do so because they have been instructed to by an adult.

This highlights the weakness of any number tasks executed in domestic and/or pre-school settings in which children may be required to 'count' or respond to 'how many' with single number-word responses. It similarly highlights the weakness of any extended exercises to demonstrate and practise the counting procedure, as these are not purposeful activities which prompt communication about cardinality with a genuine purpose, which meets any functional need or which makes meaningful sense to 3 and 4 year-olds (Donaldson, 1978).
In the absence of children's use of numerals for cardinal number in this study (compared to other uses) we see

instead the 'personalised' ways in which the pre-school children in this study represent quantity. As Hughes (1986) found, many children create their own meaning for symbols based on their individual conceptualisations. In this way, superficial mathematical procedures are adopted without a full understanding of underlying concepts and meanings.

Such individual forms of representation in the three cardinal number tasks suggest that the minimal use of spoken and written numbers may reflect limited conceptual content (Sinclair & Sinclair, 1986). Although they may well be able to write the numerals when directly asked, participants did not spontaneously use them in the tins game. Although they may well be able to respond to 'count' or 'how many?' if directly asked, they did not use numerals in their speech in the fast food task. This would endorse the findings of others that the meanings young children hold for cardinal number is limited (Cowan, 1987; Gelman et al, 1986; Dowker & Cowan, 1998).

As we have seen, caregivers in the home rarely provide 'last number word repetition' or acknowledge the cardinal aspect of the set, nor do they link number words with nouns, i.e. the names of items counted, a practice which would give meaning to counting and cardinal processes. This socio-cultural practice appears to be directly reflected in the low scores in the cardinal number tasks. After all, if caregivers do not model or explicitly explain the cardinality concept alongside the names of the objects (e.g. "three cats"), a child cannot observe 'actions on referents', such as objects, and record or parallel this action with a symbol as suggested by Hiebert (1988). If this does not take place young children may attribute last number word repetition instead to 'task completion' (Fluck, 1995) and find cardinality a difficult concept to grasp.

If, as Vergnaud (1982) suggests, participants' 'grasp of concepts is reflected in the way they do or not make use of symbols', then the findings of these tasks suggest that young children understand even less than might commonly be expected about the use of cardinal number for such everyday purposes as communicating quantity to others, either verbally or in writing.

7.2 Non-cardinal uses of number

As there has been little research into children's broader understanding of number and numerals, i.e. for uses other than cardinality, since the work of Sinclair & Sinclair in the late 70's and early 80's, the other two tasks in this study were aimed at exploring and adding to this evidence base in a 'broader and more comprehensive' way (Cowan, 1991). As the study aimed to uncover some of the meanings that participants held for number, a wider range of uses were used, some cardinal but also nominal and ordinal, and a range of different contexts.

The third study involving the Magic Book examined children's awareness of 'environmental number', that is their awareness of the presence of printed numerals in the child's everyday environment such as on labels and items in regular use, and of the meanings of these numbers. The aim was to examine participants' grasp of number concepts related to environmental applications reflected in the way they did and did not make use of symbols (Vergnaud, 1982).

Participants were most aware of the presence of numerals on the telephone whereas all found the coin the most difficult. Generally there was a very close relationship between the ability to say what was missing and their understanding of the purposes of the numerals, although English children were better at explaining all items and a little better (but not significantly so) at entering the missing numerals on the pictures.

The findings therefore show that children in all three countries were developing a better grasp of numerals when these were used for purposes other than cardinality. This mirrors the findings of Sinclair & Sinclair (1984) who also found that responses varied considerably with type of item. They found that the easiest items for children to refer to were a cake showing an age-number and the floor-number buttons in a lift. Taking again the socio-cultural view, Sinclair & Sinclair propose that this is due to the presence of visual clues provided by the context, i.e. as there were five candles placed on the cake, and there were five buttons in a row on the lift panel.

It may well be the case that the visual context supported children's responses in the Sinclairs' study, but in this study the items shown in the Magic Book were shown in isolation and out of context and not with any environmental surroundings. The Invitation card could not suggest the purpose of the numerals, as there were no visual cues for the children as they responded to the questions asked about the date, time, address and phone number.

Responses to the coin in the Magic Book are interesting to consider in view of the problems involved in participants recalling and explaining the number. Although the numeral on a coin is a cardinal number (in the sense that a 5, for example, means five pence, krona or yen) it is possible that a young child may attribute the 5 with a nominal meaning or name, for several reasons. Firstly, in the domestic or nursery setting the young child is often asked "what's this?" as if 'five' is its name or identification, or asked to give "the 5p/five pence". Secondly, a child cannot readily see evidence of its cardinal meaning, i.e. they cannot see and count five single pennies as only *one coin* can be seen. Thirdly, the child may rarely accompany an adult on shopping trips, the adult may frequently pay with a card so no coins are handled, or cash may be handed over without any explicit reference by the caregiver to the coins, notes, their cardinal number, equivalence or value.

Socio-cultural factors would therefore appear to account for both participants' difficulties with the coin, which they found problematic, and for a contrastingly stronger grasp of the numerals on the telephone which they found easiest to explain. The telephone is an item readily seen in use in the home, as also reflected by the *rate* of response to the phone number question on the Invitation task (even though less successful at providing strings of three or more digits).

The findings of the Invitation task showed that stating the time and date were found easier than other items, with no significant findings for the address. Giving the time may be seen as involving fairly complex or unusual use of language ("o'clock", "half past") and difficult concepts i.e. measuring the passage of time, the meaning of analogue and digital clock-faces, and the varying sentence constructions for time. Nevertheless this was the most successful question.

Why should this be? If we take the socio-cultural stance, we might consider that time is used more often in the environments in which these young children spend their time. Where time-keeping is important to adults, as would apply to these young children who have to be taken to school, and as they participate in time-tabled activities during the nursery-school day, they may well hear clock times mentioned quite frequently.

The date would perhaps seem less likely to be found easy, as children in nursery settings are not often required to read and write the date. However the context used in this study, that of a child's birthday, may be the context in which young children are most likely to hear about dates. They may also become aware, if they receive an invitation, of the dates of parties which they attend or are invited to.

On the other hand, the telephone number might initially be considered easier for children to respond to, as it involves straight-forward repetition of number names with no special sentence construction or vocabulary. Furthermore, in the invitation task only three or four numbers were required when giving the telephone number, yet this is the item found the most difficult. Perhaps it is the case that we rarely ask young children their telephone number as we are more likely to ask adults or older siblings. Also, repeating or learning a sequence of three or four unrelated digits may be something that we would not ask young children to do. Three and four year-olds do experience repetition and 'learning by heart' but this applies to rhymes and actions etc. which generally hold interest and meaning for them.

There were no major differences between countries, which is perhaps unexpected considering their different cultures. Most interesting perhaps is the comparison with Japan. This is a country in the Pacific Rim that has received particular attention for the high scores in mathematics achieved by children aged nine and above, in both the TIMMS studies and the OECD study (2001). As the results of this study show, despite an extensive programme of well-established nursery education, and despite differences in the culture, Japanese children do no better at ages three and four years.

However, as shown above the results appear to endorse the theory that the extent and conceptual content of young children's understanding of cardinal, nominal and ordinal number results from the quality and degree of socio-cultural support for their learning. Across all three countries this support seems to be most prominent in their everyday lives and principally in ordinary, domestic and community settings, rather than in school, and for purposes other than cardinality.

7.3 Apprenticeship to the language and conceptual frameworks of number

The tasks of the Milk Note, Magic Book and Invitation were designed considering the possibility of comparing learning about spoken and written number with the process by which young children come to learn to speak, read and write words and text. All items in the Magic Book and Invitation tasks used printed materials taken directly from the child's everyday environment, and all involved the writing of numerals as well as words.

A comparison may be made referring to Yetta Goodman's recommendations (1980) for developing 'literacy roots' - as has now been achieved through the well established curriculum for early literacy (QCA 2000; DfEE 1998). According to this approach the child is viewed as an apprentice to language and literacy (Miller 1977) when real life and spontaneous examples of writing for meaning (Goodman 1980) are used to convey meaning in messages to others as a form of communication. Here the adult acts as a scribe to record in print the young child's memorable speech (Rosen 1989) which holds meaning for the child as he reads it back, thereby avoiding writing 'exercises' or what Hall (1987) describes as the tradition of meaningless drill.

In the light of the findings of this study, in particular those showing that children develop less understanding of cardinal number and numerals, we may question whether children's everyday experience provides a similar cognitive framework for number. Most importantly whether they hear the 'language of number' to the same extent as they hear the 'language of print' (Goodman 1980, Wells 1981). Certainly in Munn's (1994) work with pre-school children she did not find any such similarly stable framework for counting. The findings of this study also call into question whether this is the case generally with regard to wider aspects of cardinal number.

In the case of school mathematics it is frequently reported that children often adopt symbolic and algorithmic procedures without an understanding of underlying concepts (e.g. Cockcroft 1982, Resnick 1987). Vergnaud (1982) recommended that these are best assessed by examining children's ability to solve ordinary problems using 'natural language'. This is what the tasks in this study aimed to do, revealing participants' misunderstandings and limited knowledge, most particularly when spontaneously communicating about cardinality and when using associated number symbols. These findings further endorse Hiebert's doubts children's competence with written symbols (1988).

The findings also appear to support Durkin's claim (1993) that before starting school children rarely undertake counting because they want to know how many objects there are, as they only count when directly asked or instructed to by an adult. Furthermore they do not see the use or power of spoken and written number for communicating information about quantity to themselves or others. This supports Sinclair & Sinclair (1986) and Hughes (1986) who observed a split between children's own representations of quantity, based on one-to-one correspondence and concrete objects, and teaching in schools which often begins with identifying the names and order of numerals and the conventional system of numerals.

It may be that as Sinclair & Sinclair suggested (1986) few links are made by the school between formal symbols and children's personal meanings because schools severely underestimate the conceptual content of the symbol system. A similar situation appears to exist in the home where caregivers' do not explicitly teach cardinality because they assume that their children already understood the purpose of cardinality Fluck (1995). This may also be the case in educational settings for 3 to 5 year-olds, but this has yet to be identified or substantiated in research findings.

The Ofsted Report published in November 2001 comments on the teaching of numeracy in the reception classes of 129 schools inspected during the previous year. The report describes the quality of the majority of numeracy teaching as satisfactory or good, with implementation of a 'full' numeracy lesson by the middle of the summer term. The report does not provide the level of detail relevant to this discussion, but my own observations in reception classes in the East Midlands and Lancashire, also over the past year, suggest that emphasis on the purposes, meanings and communication of cardinal number is not commonplace in the classes of four year-olds. However it is interesting to note the findings by Christ Church University College (Aubrey, November 2001) that, despite the early school start in England, the mathematical skills developed by age nine years are no better than those in five other European countries where children start school at ages six or seven.

One project in the South-East Midlands is monitoring the progress of their children against the QCA's Early Learning Goals at the end of the Foundation Stage (Waller 2001). The results are prompting widespread concern locally about the scores in mathematical development which are significantly lower, and showing significantly slower progress, than those obtained in literacy and other areas of development. This suggests there is a valid reason for investigating the nature of the socio-cultural and subsequent educational support for early number development.

7.4 Conclusion

Comparing the cardinal and non-cardinal results seem to suggest that innate factors are unlikely to be the only source of young children's limited use of cardinality and relatively better grasp of other meanings of number. The findings also indicate that understanding of number is not pure knowledge independent of humans. Young children have to discover what adults mean concerning counting, cardinal and other forms of number in order to make the meaning their own. This appears to further endorse the socio-cultural view and support the case for examining the social mechanisms of cognitive change in young children and how they acquire meaning about number.

Alongside the challenge of discovering and uncovering meaning, children have to learn the pragmatics of language in relation to number. Little attention has been paid to development of the linguistic-cognitive underpinnings of early number when conceptual frameworks are being formed, or the language and linguistic structures used by adults in the course of common number activities in the home and pre-school.

The findings of the study cast some doubt on the nature and amount of the early support for, in particular, cardinal number development in the home. The findings also raise questions about associated educational practice in nursery and reception classes for three and four year-olds. It follows that the grasp of number that three and four year-olds' constitutes a weak foundation on which to begin or extend teaching about number and number symbols when children enter the education system at ages four, five or six. It is also a flawed basis on which to commence the teaching of practical addition and subtraction as recommended in the curriculum guidance for 3 to 4 year-olds in UK nursery settings (QCA, 2000) and 4 to 5 year-olds in UK school settings (QCA, 2000; DfEE, 1999). It has been proposed that the conceptual content of the symbol system is severely underestimated by schools and assumed by parents.

There appears then to be an urgent need for children's difficulties, particularly with regard to cardinal number, to be re-examined in relation to their understanding of when symbols are written by others and when children need to use symbols themselves. This is particularly important in view of longstanding evidence that early difficulties make it both difficult to engage children (Wearne & Hiebert, 1989; Elton report, DES 1989) and to correct difficulties later (Resnick & Omanson, 1987; Nunes & Bryant, 1997). It is also well documented that such difficulties soon become compounded, resulting in further delay (Hadow Report, 1931; Cockcroft Report, 1982; ALBSU, 1992). The findings of the study would seem to predict that subsequent teaching of formal symbols without meaningful referents results in learning difficulties at a later stage, as Hiebert (1988) and others describe. The local UK study (Waller, 2001) already indicates a delay in early mathematical development by age five.

In 1989 Perret-Clermont called for the socio-cultural mechanisms by which children acquire number meanings to be examined, supporting Puttnam's call (1989) for the need to look beyond purely cognitive processes for the referents of (number) words. However there has yet to be undertaken a comprehensive and detailed examination of the socio-cultural contribution to early number development in similar depth to that exploring literacy development (Goodman, Waterland, Wells, Hall and others). An investigation is also long overdue into the influence of socio-cultural support in *relation to innate cognitive factors* in children's number development at this age; this includes linguistic support, children's oral responses and the generative role of language in cognition.

Equally important, I would suggest, are the negative effects of incomprehension and lack of mastery of a prominent part of the curriculum in which young children are involved on a daily basis. It may even be the case that such incomprehension may prompt the early lack of confidence in mathematics that becomes exacerbated by further failure during and beyond the later school years.

The question may be raised as to whether, during infancy and the pre-school years, young children become 'apprenticed' to the different meanings and uses of number in the same way that they learn about the uses and meanings of language, reading and writing during the early years. The findings of the study suggest that this may currently not be the case.

Chapter 8

Reflections on the current study

Overall I feel that the study has provided a valuable opportunity to collect data in two other countries, Japan and Sweden, that might otherwise have been very difficult to collect. Certainly it would not have been possible to carry out the testing in Swedish and Japanese classrooms in the context of a normal nursery school day (ensuring the children felt relaxed) without the collaboration of experienced early years practitioners who were native speakers. In the case of Japan, it would have been impossible without the official permission to work in local schools with a large number of children given by the education authority in Osaka District. Furthermore, such a large study would otherwise have been impossible for me to carry out working with the resources at my disposal.

It is hoped that the comparative data on children aged three to five years will contribute to knowledge of children's cognitive development in the domain of mathematics. In the area of number development the data may extend the information we already have on what very young children are able to learn and understand about cardinal number, and in particular on how they verbally express cardinality and how they use symbols. I also hope it will broaden the knowledge base in psychology by adding to our understanding of when and how young children develop in areas of number other than cardinality.

Overall it may eventually contribute to our knowledge of the relative roles of the physical, social, linguistic and educational environments in young children's mathematical development and how these interact with the young child's innate capacities. I would be pleased if this study prompted further research in these areas.

The comparative data for this age-group may also contribute to the current educational debate in the U.K. over 'standards of attainment' in mathematics. Whereas international studies such as TIMMS and OECD provide data on children aged seven and above, there is relatively little data available on children of pre-school age against which to compare the educational starting points of children in any one country.

The data may further contribute to the broader picture by adding a longitudinal perspective. As both of the non-U.K. countries in the study have a history of pre-school education and as one of the countries is in the Pacific Rim, popularly attributed with high maths standards among older children, the findings may provide interesting comparisons of children's progress over time.

Carrying out a comparative study was not without practical complications and problems to overcome. Firstly, the instructions and recording sheets for the tasks had to be translated into the Swedish and Japanese languages and into Japanese script, all of which were complex and time-consuming activities. Secondly, some tasks had to be changed, removed or adapted to suit the cultural situations of the participants, as indicated in the Method section.

For example, in the Fast Food task the food items shown to Japanese participants were changed to be culturally relevant to ensure children's engagement with the task, and safeguard the validity of the test. In the Magic Book, the items that were photographed were changed in each country so that they would be familiar to the participants, e.g. English, Swedish and Japanese telephones, an English and a Swedish bus and coins showing denominations of pence, yen and krona.

Thirdly, steps were taken before leaving England to aid the administration of the tests by planning a training programme for the research assistants in Japan and Sweden. In view of the findings of recent research which shows how sensitive young children are to adult language and interaction, it was considered important to closely control the actions and language used by the research assistants during the testing process. For this purpose a video recording was made of myself carrying out each of the tests for the purpose of reliability. On the video I was shown working with two children, a less competent boy and a girl who was more confident, in order to demonstrate the way I conducted the tests, used the materials and responded to the children's responses and queries.

In Japan I was present in the nursery classrooms throughout the testing period but in Sweden I was not able to be present in the schools due to work commitments. The period of data collection in each country appeared to run smoothly.

Naturally there were some drawbacks with planning to work with research assistants, children and teachers who were initially on the other side of the world and from a different culture. Few difficulties were apparent with administration of the tins game, which appears to have been conducted reliably in all three countries. This may have been due to the fact that it was the first test to be demonstrated during the training sessions and the research assistants were fresh. The actions and questions were repeated several times on the video as all four tins were shown, so it may be that this degree of reinforcement proved helpful for the assistants when training.

The Milk Note task was adapted for participants in Japan and Sweden who do not have a milkman deliver milk to the house; they were asked instead to write a note to the shopkeeper to ask for two *cartons* of milk. On reflection it may have been advisable in the circumstances for the research assistants to firstly show photographs of the shopping situation and discuss with children that the shopkeeper needed a note which says that two cartons of milk are required. It may therefore also have been advisable to provide Japanese and Swedish assistants with a script and pictures to support their explanations, and to include a check that participants had understood the aim of the task. This is something that I recall I did myself spontaneously, but it did not feature in the research design.

The Fast Food task was also demonstrated on the video for discussion by the research assistants and at the time there did not appear to be a problem with administration. However the Swedish results later raised questions about how the (fast food) task was explained to participants. This task was devised on the basis of children's familiarity with obtaining food from a restaurant or take-away, however in retrospect it could be reasoned that we cannot thereby conclude that the children were familiar generally with conveying information on the telephone, although the results on the Magic Book suggest that participants understood the purpose of the telephone. When replicating this task, again it may be advisable to provide explanatory statements and photographs for assistants and to require them to check that participants understand the purpose of the task.

The results of the Tins Game, Milk Note and Fast Food tasks appear to suggest that activities involving cardinal number are novel to young children. In this case greater explanation of the three tasks may have been required. It may be that some tasks were less appropriate for replication in the other countries (e.g. the milk note and fast food tasks) whereas specially designed tasks may have been more reliable. This will need careful consideration and discussion with others in the relevant countries.

It remains unresolved whether the low levels of successful responses across all three countries could be due to factors other than, or additional to, participants' grasp of cardinal number. In future tests, as far as possible it would be essential to rule out any difficulties arising from research assistants' practice. This has implications for the training of the research assistants, particularly in comparative studies where long distance, cross-cultural arrangements have to be made across three or more languages.

It is interesting that the results of the Invitation task suggest that it invoked far fewer problems for research assistants and participants. This may have been for three reasons. Firstly because the Invitation card held intrinsic meaning and purpose for the child, decorated as it was with balloons and party items; secondly because each question written on the Invitation was simple, unambiguous and was read aloud to the child; thirdly because in addition to what the participants wrote, their oral responses were taken into account.

Participants also scored quite well on the Magic Book task which had each question scripted in simple language for the assistant to read aloud to the child, and had photographs and pictures to be shown. Other than this no additional explanation by the research assistants was required. One drawback with the Magic Book was the different items shown to participants in the three countries, changed at the last moment in response to the Japanese assistants. This was unfortunate as it reduced the amount of comparative data and prior agreement on all four items would have been beneficial.

It was not until the latter stages, when extracting data from recording sheets, when and translating from Swedish and Japanese, that any of the above anomalies arose. As I was unable to closely monitor the research assistants' use of language during data collection in the other two countries due to language differences, there may have been some misunderstandings among research assistants of some aspects of the tasks that I was unaware of. For example, in Japan the assistants failed to collect data in the tins game on how the children read back their labels to recall how many blocks there were in each tin.

It would therefore have been beneficial to have had a translator on hand when each phase of data collection was underway. This would have allowed for checking of the recording sheets to ensure that the handwriting, script, sketches or jottings could be easily read and understood and allow for any omissions or corrections to be attended to in situ. This would then ensure that, on return to the home country, the records of participants' responses were unambiguous when later extracting the data. However this also has implications for the allocation of additional time and personnel in the field at the end of testing period.

Finally, there are some changes I would wish to have made even to the English study. Firstly, in view of the fact that the validity of the findings of all tasks rested upon children's grasp of numerals, it would have been advisable to pre-test for participants' knowledge of how to read and write numbers, at least 1 to 4. Although this was carried out informally with the English participants, all of whom were successful, it should have been built into the original research design and formed part of the recorded data in all three countries.

Secondly, in the fast food task it may be have been more reliable to have used a second person to receive the food orders that were given orally, to act as an audience for the children's communications. Of course this would increase the number of research assistants required, or if they worked in pairs on the tasks, the testing period would become longer.

Finally, I would like to have used an amended version of the tins game as an additional and final task; in the amended version the interviewer would serve as the audience who would read back the labels. This would have allowed an examination of whether 'writing for another person' effected a change in the types of representation used by participants when recording cardinal number in graphical form.

In view of the difficulties of administration, perhaps such a project needs to be financed in order to allocate greater time for the training of research assistants, to have a translator on the spot to monitor the testing process, and for post-checking. Furthermore, although the assistants involved in this study were experienced in working with pre-school children, they were not trained in the methods of psychological research. If the project *had* been funded it may have been possible to recruit and train assistants who also had experience in developmental psychology to carry out the data collection, making a further contribution to the reliability and validity of the tests.

Chapter 9

Recommendations for research and education

As the discussion in Chapter 8 suggests, further research is needed in the longer term to investigate particular aspects of the findings and to explore broader issues concerning the development of children's understanding of number. Recommendations can also be made in the short term for additions and improvements to current educational guidelines, staff training, classroom practice, working with parents, educational materials and software.

Recommendations for follow-up studies within child development and proposals for intervention studies are discussed below.

9.1 Further research on socio-cultural support

The principal question arising from this study relates to the nature of socio-cultural support provided for the infant and pre-school child with regard to early number concepts and representation. This would encompass its wider effects on the young child's linguistic development and social cognition in the domain of early number.

9.1.1 Adult scripts for the language and uses of number in the home

The findings of the study differ according to the uses of number, i.e. cardinal, nominal, ordinal, which may reflect differences in what the young child has experienced in the course of everyday life. Currently we have little or no information on this matter. My own informal observations suggest that many adult activities involving number are undertaken in a solitary manner, mentally and silently, i.e. without the involvement of another adult or a child, thereby denying the young child a model of the linguistic script, relevance and use of number, i.e. not demonstrating how number is used as a form of communication to oneself and others.

One possible study would examine everyday interactions in the home environment. A similar study was undertaken by Gordon Wells in relation to the learning of language and literacy (Wells, 1985) and it may be fruitful to examine the content and nature of adult-child interactions on the audio-tapes and transcripts arising from that study.

However, a further longitudinal study is needed if it is to be more culturally relevant to contemporary society. Such a study would provide quantitative and qualitative evidence on the nature of everyday mathematical tasks observed by the young child, those in which he might participate or might initiate, the scripts (Nelson, 1973) of number activities, number language and associated linguistic structures.

The subsequent analysis would provide information on when and how young children have modelled to them how last number repetition represents cardinality and whether nouns are used alongside cardinal number words. It would also provide data as to whether cardinal number tasks are undertaken in the context of normal everyday conversation as joint activities between family members or principally as educational 'exercises' created to instruct the child.

The analysis would also provide information on the contexts and tasks normally undertaken in the home which involve number. It would provide us with information as to whether oral and written communication by adults are a regular feature, and the extent to which the child is exposed to these experiences, both orally and visually. Such a contemporary study would provide information on children's experience and awareness of pen and paper activities which involve number alongside use of the telephone and electronic means of representing number and text such as in games, television and computer based internet facilities, on-line banking and shopping, use of the palm-top organiser and calculator etc. This would provide data on the overall balance of number uses made available to the child, cardinal, ordinal and nominal.

9.1.2 How children ascribe meaning to number

This study would consist of a series of individual case studies involving young children in observation and semi-structured interview. The observations would take place in play settings created specifically to prompt, as far as possible, use of number by the child (shopping, kitchen play, sharing out food or toys etc.).

Firstly the settings would allow for structured observation and coding of the types and frequency of the child's *spontaneous* use of oral and graphic forms of representation and communication, i.e. not in response to adult questions.

Secondly, follow-up interviews would explore reasons given by the child for her actions, speech and representations in the observed activities. These responses would then provide the basis for probing and investigating the meanings assigned by the child to the different uses of number, cardinal, ordinal and nominal.

The interviews could develop into a longitudinal study of children's development within and across the different uses of number, investigating changes with age. This study could be allied to an intervention study to examine the effects of different adult practices (see below). Manipulation of the data could allow comparisons between individual cases to uncover any common patterns in:

- the types and frequencies of the meanings that young children hold for numbers used in different situations within the everyday environment (nominal, ordinal, cardinal, time, money, measures etc.)
- the prevalence of oral and graphic communication involving number among young children according to the medium (face-to-face, paper, telephone, electronic)
- the nature and forms of linguistic structures used by young children as they communicate orally with others in situations involving number
- the nature and forms of graphical representation used by children for their own purposes and when prompted to communicate with others
- the ways that adults and children negotiate meanings and develop common knowledge (Edwards & Mercer, 1987) about number and in which situations
- the generative role of language in mathematical development

Some sections of the methodology and tasks used in this study may be appropriate for these purposes, such as the revised Tins game and Milk Note tasks, discussed in the previous chapter, in which a recipient or audience is provided for acts of oral and written communication.

9.1.3 Scripts provided by adults in pre-school centres

Projects similar to those for the home, discussed above, could be established in pre-school settings. The settings could be both educational and care-based, with vocational, unqualified and professional adults catering for two, three and four year-olds. The projects would also involve structured and coded observations of adult-adult and adult-child interactions, designed to uncover the types of number experiences, language and scripts to which young children are exposed.

The added variable in these projects would be to investigate the mathematical content of planned activities alongside that found in the informal domestic activities which form part of the pre-schoolers' day. Appropriate coding on observation schedules would allow a possible comparison between the frequency and linguistic content of formal and informal activities, and the interactions and scripts of the different categories of adults.

9.2 Adult knowledge about children's number development

Previous research has shown that following a count caregivers in the home commonly do not undertake last number word repetition or make explicit the purpose of cardinal number because they assume that their child already understands the concept of cardinality. Whether last number word repetition is regularly demonstrated in pre-school settings is not known, which suggests that further research in both settings is required of we are to offer greater support for young children.

9.2.1 Pre-school staff knowledge about number

Earlier research has shown that pre-schoolers often find counting to be purposeless. Bearing in mind that language development is founded on the child's need to communicate, it would appear essential for the adult to understand what the child already knows and understands about communicating number, both orally and graphically.

A further project would involve interviews with the pre-school staff at all levels concerning their grasp of the principles of cardinal number and how much they believe pre-school children know and understand about the meanings of number. Where possible this would follow up the structured observations discussed above, for the purpose of cross-referencing.

9.2.2 Caregivers' knowledge about early number

At present there is little information on how much caregivers know about children's number development. This project would involve use of a questionnaire and would complement the longitudinal study outlined above by also providing quantitative data on the views of parents and caregivers as to the understanding and skills of their children.

The findings of this study could provide the basis for the study by asking caregivers to predict the outcomes of such research, i.e. by stating what they believe children aged three and four years understand and are able to do.

The questionnaire could probe further the question of whether caregivers assume children's grasp of cardinality as well as exploring their assumptions concerning the understanding of times, dates and uses of numbers in the environment such as on phones and coins.

9.2.3 The effectiveness of adult support

A further way of investigating the data would be for participants' responses to be matched against the material provision, adult practices and language observed in the participants' pre-school settings. This would allow an analysis of those settings and adults in terms of the characteristics of socio-cultural support and professional knowledge which are particularly effective in scaffolding and developing an understanding of both number meanings and the use of representation and communication. The same could be undertaken regarding the support received from caregivers in the home.

9.3 Learning the meaning of other mathematical symbols

In view of the findings of this study and in the light of the common and longstanding difficulties encountered by many children and adults with symbolic aspects of mathematics, similar studies could investigate the grasp of other symbols.

Adults continue to experience problems with arithmetic, and children and adults struggle with certain aspects of measure. It has long been the assumption that it is the nature of the calculations that is the cause of the problems. However, as this study has shown, there exists the possibility that the underlying problem may lie in an inadequate or erroneous grasp of the meaning of the symbol which they are required to work with.

Further research could be carried out to investigate the meanings that learners hold for the equivalence or 'equals' sign as this is known to be problematic, particularly when 'missing number' problems are presented in arithmetic and beyond in algebra. The full range of meanings of the four operator signs for addition, subtraction, multiplication and division would also be valuable to investigate.

The author has undertaken pilot studies with seven year-olds in written arithmetic, and with five and seven year-olds in measuring with a centimetre ruler, and these studies have provided interesting data on the erroneous grasp of the meaning of such symbols.

9.4 Intervention studies

The similarity in findings across the three countries and cultures raises questions about the socio-cultural support for young children in the different areas and uses of number. However we can only conjecture as to the contribution made by innate factors and the relative contributions of others aspects of the child's development. Furthermore we can only hypothesize about the effects of support in the home and nursery class, and what the child might have achieved if the support had been different.

Intervention studies with carefully controlled groups may go some way towards providing such information.

The content of the interventions would be based on number activities designed to hold interest and meaning for participants, as discussed in this study. They would also need to provide genuine purposes for using and communicating number to themselves or others.

The studies could involve cardinal number, as this is a principal point of interest, but it may be useful to take the other uses of number which were found difficult, such as the telephone number and coin, and provide additional experience for participants.

The aim of the intervention studies would be:

1) to investigate the extent to which young children are capable of sharing the meanings of number with adults and of participating in joint activities which prompts them to make the meanings their own;

2) to examine whether lack of meaning is the principal cause of difficulty with some of the mathematical procedures.

The study would take the model of language and early literacy research for the design of the intervention. By providing purposes which genuinely motivate young children to communicate, it can be investigated whether children are able to grasp meanings and use number orally and graphically at ages three and four.

Control groups could receive matched amounts of time of their usual activities in the mathematics curriculum. Other intervention studies can be designed in relation to arithmetic and measures in which the language and meaning of the symbols are directly addressed in the intervention. The author has carried out an intervention with seven year-olds all of whom misunderstood the zero and number symbols on a centimetre ruler. The intervention was brief but successful, even after a six-week lapse.

9.5 Further studies in developing countries

It is important to note that the experiments in this study were all undertaken in countries within the developed world, with children receiving nursery education. As the findings appear to suggest, the nature of the support provided in the home and nursery class may not be sufficient to prepare young children for using the language and symbolic forms of representation that are common knowledge among many adults.

Reasons for this have yet to be fully investigated, but as a part of this follow-up investigation, further intervention studies could take place in both developed countries and developing countries.

While developing countries may experience the problems of poverty and lack of formal education, there exists the possibility of investigating the nature of the young child's extended experience of number in the towns, villages and domestic settings of less developed communities. Where the adults have not received formal education they are less likely to use school-type methods in the home. These adults may also be less likely to use 'privatised' mathematics (i.e. not shared with others) but more openly share activities and goals that involve practical, informal mathematics, both with children and with other adults.

An investigation into such practices, combined with interviews with three to five year-olds on their understanding and communication of number, may provide further data with which to expand our understanding of the potential and capabilities of the young child.

Programmes of teaching and intervention studies may also investigate the effectiveness of alternative teaching methods in preventing the types of problems encountered by the participants of this study, i.e. by avoiding development of the cognitive difficulties that appear to underpin them.

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